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ABSTRACT

This workbook serves as a self-learning guide to basic mathematics and treatment plant calculations and also as a reference and source book for the mathematics of sewage treatment and processing. In addition to basic mathematics, the workbook discusses processing and process control, laboratory calculations and efficiency calculations necessary in all steps of sewage treatment plant operations. The appendix includes tables of English and metric measurements, processing and dimensional formulas, and a problem answer key. (CS)

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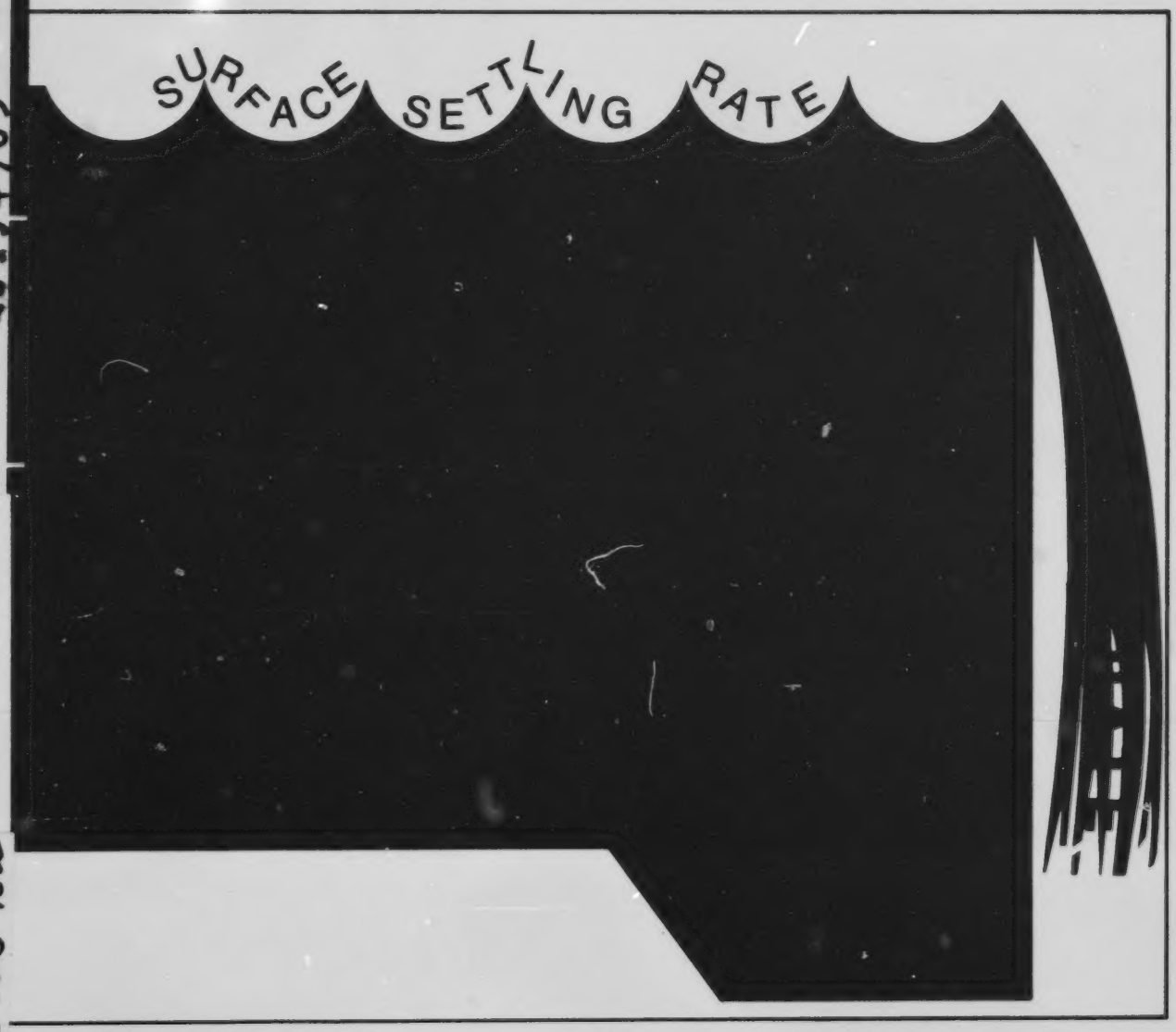
WORKBOOK

BASIC MATHEMATICS & WASTEWATER PROCESSING CALCULATIONS



ED154983

SE 023 782

WORKBOOK
BASIC MATHEMATICS
and
WASTEWATER PROCESSING CALCULATIONS

1975

Sponsored and prepared for
New York State Department of Environmental Conservation
under a grant from the
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Prepared by the

Onondaga County, Department of Drainage and Sanitation

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Preface

The inability to use basic mathematics is a common problem and is not confined to the wastewater treatment field. It is of serious concern to us because it severely limits the operators' ability to control and evaluate the treatment plant processes.

If this workbook is to have value in training or retraining operators in mathematical skills, it is essential that it overcome the reason why they did not learn math in the first place. With this in mind, let us consider the whys of poor math performance.

Far too often the teaching of mathematics has been an exercise in rote memorization and repetitive exercises with rigid requirements that the student-adhere to the teacher's set procedure. More often than not, it has been limited to a meaningless manipulation of numbers poorly related to everyday use. The net result has been to make it a skill to be avoided, feared and used as little as possible.

For these reasons, this work is to be presented as follows:

To explain each mathematical procedure in as simple, clear, and concise a manner as possible.
To present a detailed example and solution to each procedure.
To provide realistic practical problems and keyed answers for practice and to establish confidence.
To provide review sections for the student to measure his own progress.
Each basic mathematical procedure is to have a meaningful application to sewage treatment.
The processing calculations sections are to be preceded with a short description and explanation of the meaning and use of the calculations.

It is the intent that this workbook serve, first of all, as a self-learning guide to basic mathematics and treatment plant calculations, and, secondly, as a reference and source book for the mathematics of sewage treatment and processing.

This workbook applies to processing and process control, laboratory evaluations, and efficiency calculations. It is not within the realm of this project to cover the areas of hydraulics, electricity, and maintenance mathematics for which there is a definite need.

The numbers presented in the operational discussions are general and are not intended to apply to all plants and processes. They are ballpark figures. Operating figures must be determined for each plant design and will vary with the sewage being treated.

RICHARD R. METCALF
Project Director

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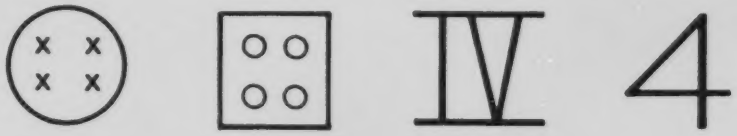
NUMBERS & NUMERALS

What are numbers?

Numbers are an idea or a concept.

Numerals are symbols which we use to represent or express numbers.

The number four (4) can be represented in many ways as shown below:



The symbols or numerals we use in our mathematics are called the Arabic system of numerals.

This system has ten (10) different numerals to represent ten (10) different ideas or numbers or quantities.

0 1 2 3 4 5 6 7 8 9

Each progressive number contains one (1) more quantity than the preceding number.

5 is one more than 4

9 is one more than 8

1 is one more than 0

When we want to represent a number that is larger than nine, we use the basic numbers in combination. This gives us numerals of more than nine (9).

10 gives us 1 more than 9

11 gives us 1 more than 10

12 gives us 1 more than 11

13 gives us 1 more than 12

and so on.

When we want to represent a number larger than 99 we use the following combination:

100 gives us 1 more than 99

101 gives us 1 more than 100

etc....

The first column of numbers is called the ones column such as:

1
7
6
5
9

all in the ones column

When we get a number with two places, such as 12, we then call the column to the left of the ones column - the tens column.

tens	ones
1	6
2	4
7	5
3	2

When we exceed 99 we then move to the left of the tens column to the hundreds column.

hundreds	tens	ones
1	4	7
5	2	6
7	3	8
9	1	1

And when we exceed 999 we move left to the thousands column and so on.

This system will make up a picture as below:

Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
6,	3	0	2,	4	1	6

So with only ten (10) numerals, we have worked out a system of numbers that could go on indefinitely.

Example

748

How many hundreds in this number? 7

How many tens in this number? 4

How many ones in this number? 8

So to read the number it would be seven hundred and forty-eight.

The numerals we use are also called digits.

Arranging any number of digits in a side by side arrangement gives us numbers of different quantities.

7,395 = seven thousand-three hundred-and ninety-five

The same digits in another arrangement would be a different quantity.

3,975 = three thousand-nine hundred-and seventy-five

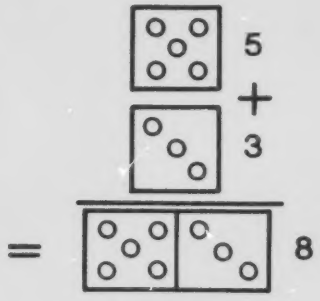
The subject of mathematics is the study of the manipulation of these numerals or digits which have been used to represent numbers.

There are only four basic procedures of numerical manipulation. Each procedure has a symbol which represents that procedure.

Addition	+
Subtraction	-
Multiplication	x
Division	÷

ADDITION OF WHOLE NUMBERS

The addition of 5 + 3 can be explained this way.

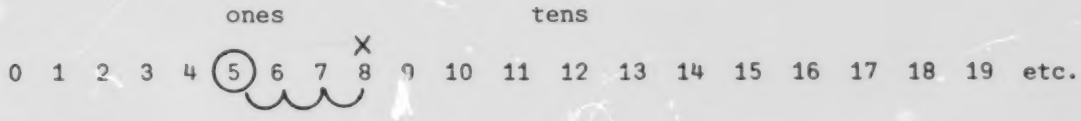


When we add these two boxes (or join them) and count the dots we have eight (8).

Another way to visualize this concept is to make a chart of our numerals.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 etc.

The above problem 5 + 3 would be solved like this:



Start with the five (5), then move over to the right three (3) digits and you end up with eight (8).

By using this chart, we can solve simple addition problems.

Do the following problems with this chart:

1) 1 +9 2) 7 +8 3) 1 +8 4) 8 +5 5) 2 +6 6) 3 +5

When the sum becomes larger than the ones column can handle, we move over to the tens column to the left and start to put numbers in that column.

This also happens in the tens column. When the sum becomes larger than the tens column can handle, we move to the left and use the hundreds column.

Using the chart add these columns:

7)
1
7
5
+3

8)
3
9
1
+7

9)
1
6
2
+9

This brings up a system of addition that speeds up addition of long columns of figures. It is called adding by tens (10s).

Example number 1)

1
7
5
+ 3
16

1 ten
+ 6 left over
16

10
+ 6
16

We pick out the combinations of digits that will add up to ten - put this in the tens column of the answer, then what is left over is added to the total number of tens.

Example number 2)

3
9
1
+ 7
20

2 tens
+ 0 left over
20

20
+ 0
20

Example number 3)

3
9
5
+ 7
24

1 ten
+14 left over
24

10
+14
24

Example number 4)

4
6
8
9
2
+ 3
32

2 tens
+12 left over
32

20
+12
32

Add these columns of digits in this manner - drawing curved lines to show the operation.

10) 6
4
9
1
+3

11) 5
4
5
6
+3

12) 9
1
3
7
+4

13) 8
5
3
7
+6

This same system of adding by tens can be used for adding columns of more than one digit.

32
341
269
785
775
+ 393
2,563

You group the digits in the ones column and you find you have twenty three. Place the 3 in the ones column. Then you take the two tens from there and put it at the top of the tens column. Next group the tens column finding thirty six. Place the 6 in the tens column and place the three hundreds at the top of the hundreds column. Group the hundreds giving you a sum of twenty five. Place the five in the hundreds column and the 2 in the thousands column. The sum of the two numbers is 2 thousand 5 hundred and sixty three.

14) 7 6 2
3 4 9
+5 2 1

15) 3 2
5 6 4
3 2 4
+ 7 1

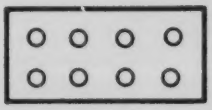
16) 2 3
1 5 6
3 2 1
+ 7 2

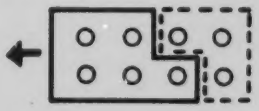
17) 6 3
1 5 2
9 5
+9 4 6

***Tip When writing whole numbers to be added, always keep the ones in a column, the tens in the next column to the left and hundreds next column to the left, etc. These will always form straight columns.

SUBTRACTION OF WHOLE NUMBERS

Subtraction of 8 - 5 can be explained in this way.

We start with a quantity of 

From this we are going to remove or "take away" a quantity of 

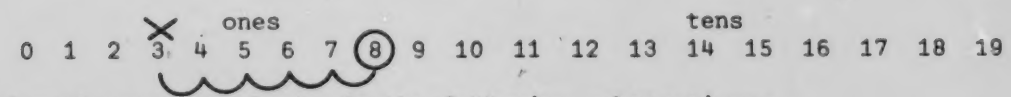
Leaving a quantity of

So 8 - 5 = 3

Using the same chart of our numerals as we did in addition, we can visualize subtraction (In addition we counted to the right).

In subtraction, since we are reducing or taking away, we count to the left.

8 - 5 = 3



Using the numeral chart, do the following subtractions:

1) $\begin{array}{r} 9 \\ -2 \\ \hline \end{array}$ 2) $\begin{array}{r} 7 \\ -4 \\ \hline \end{array}$ 3) $\begin{array}{r} 14 \\ -\ 6 \\ \hline \end{array}$

4) $\begin{array}{r} 12 \\ -\ 7 \\ \hline \end{array}$ 5) $\begin{array}{r} 16 \\ -\ 9 \\ \hline \end{array}$ 6) $\begin{array}{r} 11 \\ -\ 5 \\ \hline \end{array}$

The numeral chart we have been using does have some limitation when it comes to larger numbers. The chart would simply be too large to be practical. So we return to the column principal of ones, tens & hundreds as we did in addition.

In the problem 36 - 24 =

$$\begin{array}{r} 36 \\ -24 \\ \hline 12 \end{array}$$

we take 4 from 6 in the ones column = 2
then 2 from 3 in the tens column = 1
thus 1 ten and 2 ones = 12

In subtraction there is a problem that comes with the column system. When the number to be reduced in a column is smaller than what we are taking away, we have to borrow a ten or a hundred to use in that column.

$$\begin{array}{r} {\scriptstyle 3\ 1} \\ \not{4}\,2 \\ -1\ 9 \\ \hline 2\ 3 \end{array}$$

we borrow one ten from 4 tens, leaving three tens and use the one ten with the two ones making twelve and now we can take nine from 12 leaving 3; then we take one ten from 3 tens leaving 2 tens.
The ans. is 2 tens & 3 ones = 23.

This borrowing has to be done any time the amount to be taken away is more than the amount to be reduced in the column: ones, tens, hundreds, etc.

Working with hundreds, it is done as below:

$$\begin{array}{r} {\scriptstyle 2\ \ 11\ \ 16\ \ 13} \\ \not{3},\ \not{2}\ \not{7}\ \not{3} \\ -\quad 3\ 8\ 4 \\ \hline 2,\ 8\ 8\ 9 \end{array}$$ ans.

When you borrow always cross the number out and write in above a number one less than what you crossed out.

Subtract the following examples:

7) $\begin{array}{r} 9 \\ -3 \\ \hline \end{array}$

8) $\begin{array}{r} 12 \\ -\ 7 \\ \hline \end{array}$

9) $\begin{array}{r} 21 \\ -19 \\ \hline \end{array}$

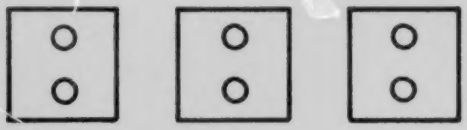
10) $\begin{array}{r} 324 \\ -\ 49 \\ \hline \end{array}$

11) $\begin{array}{r} 2{,}376 \\ -1{,}197 \\ \hline \end{array}$

12) $\begin{array}{r} 5{,}261 \\ -3{,}159 \\ \hline \end{array}$

MULTIPLICATION OF WHOLE NUMBERS

The problem 3 x 2 = can be defined as the total number of objects in three (3) sets with two (2) objects in each set.



3 sets with 2 per set

3 x 2 = 6

Another concept of 3 x 2 = 6 is that you add 2 + 2 + 2 = 6

so in turn 4 x 2 =4 + 4 = 8

or 2 + 2 + 2 + 2 = 8

Multiply the following using addition principal:

1) 4 x 3 =

2) 5 x 3 =

3) 4 x 6 =

A simpler way to multiply is to use multiplication tables which you can now develop since you know the addition principal.

0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

As you notice in the table, 3 x 4 = 12 and 4 x 3 = 12 so it doesn't matter which is multiplied by which, it always gives the same answer. (See Diagram Below)

0	1	2	3	4	5
1	1	2	3	4	5
2	2	4	6	8	10
3	3	6	9	12	15
4	4	8	12	16	20

0	1	2	3	4
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16
5	5	10	15	20

You will also notice that any number multiplied by ten, you add a 0.

In the same way to multipy by 100 you add two 00s.

Multiply these:

4) 27 x 10 = 5) 9 x 10 = 6) 14 x 100 =

7) 4 x 100 = 8) 100 x 9 = 9) 10 x 4 =

There are two different approaches to the mechanics of multiplication. Both methods (old & new) are explained here because some of you may have been taught the old way and others the new way.

Multiplication by the Old Method:

Multiplying by a single digit number

Such as:

```
  39
 x 8
----
```

1- You use the column form to show 39 x 8.
2- You then multiply 9 by 8 which equals 72.

```
 7
 39
 x 8
----
  2
```

You place the two in the ones column and carry the seven over to the tens column to be added to the answer of 8 x 3 which is 24.

```
  39
 x 8
----
 312
```

24 + 7 = 31. The one is place in the tens column and the three in the hundreths column. The resulting answer is three hundred and twelve. (312)

Multiplying a number by another number which has more than one digit.

Such as:

```
  12
x 24
----
  48
+ 24
----
 288
```

1- You proceed exactly as for a single digit multiplier multiply 12 x 4 = 48. Place the eight in the ones column & place the four in the tens column.
2- Then multiply 12 x 2 = 24. You place the four in the tens column and the two in the hundredths column.
3- You then add to get the answer two hundred and eighty eight. (288)

Multiplication by the New Method:

We also use the column system of multiplying.Multiplying by a single digit numbers.

Such as:

```
  39
x 8
----
 72
+ 240
-----
 312
```

1- Multiply 8 ones x 9 ones = Seven tens & two ones = 72
2- Multiply 8 ones x 3 tens = Two hundreds & four tens = 240.
3- Add the two giving 3 hundreds, 1 ten, & 2 ones = 312.

Using the column system & Multiplication Table: Do these examples:

10) 187 x 6 11) 752 x 9 12) 4,386 x 7 13) 5,236 x 3

Multiplying by a number with more than one digit.

Such as:

```
  12
x 24
----
   8   4 ones x 2 ones = 8 ones = 8
  40   4 ones x 1 ten  = 4 tens = 40
  40   2 tens x 2 ones = 4 tens = 40
+ 200  2 tens x 1 ten  = 2 hundreds = 200
----
 288   2 hundreds, 8 tens, 8 ones = 288
```

In this you must remember that after multiplying 4 x 2, then multiply 4 x 10 giving 40. Then you multiply 2 tens or 20 x 2 ones giving 40.

Then you multiply 2 tens or 20 x 1 ten or 20 x 10 = 200.

Placing everything in their proper places of ones, tens, and hundreds makes the addition at the end easier.

Do the following:

14) 352 x 19

15) 123 x 456

16) 576 x 309

17) 631 x 997

Murphy's Law

If there is a way to do it wrong, it will be done wrong...at the most inopportune time.

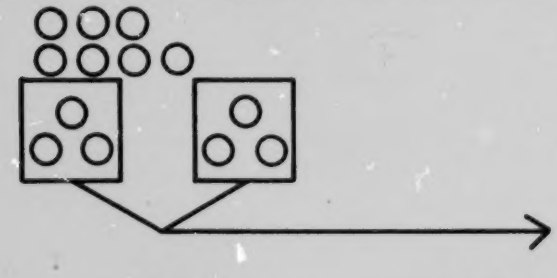
Chisholm's Second Law

Any time things appear to be going well, you have overlooked something.

DIVISION OF WHOLE NUMBERS

The concept of dividing 7 by 3, we write either $3\overline{)7}$, $7 \div 3$, $\frac{7}{3}$ to mean 7 divided by 3.

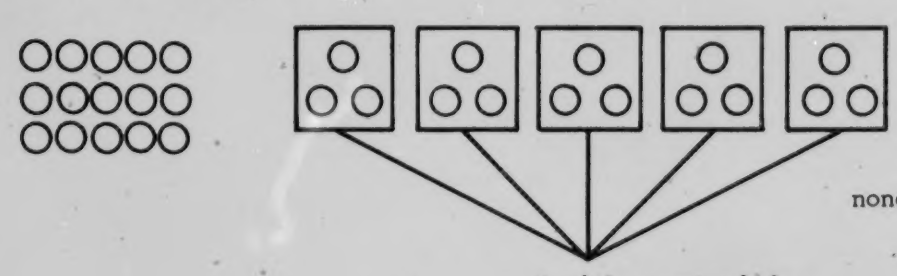
What we are asking is, if we had seven balls and we wanted to put three balls in each box, how many boxes of balls would we have?



+ ○ left over

Two with a remainder of 1

The same if we have $15 \div 3$ or $3\overline{)15}$



none left over

5 with no remainder

A division problem is set up this way $2\overline{)10}^{\,5}$

The 2 is called the divisor, the 10 is called the dividend, and the 5 is called the quotient.

The solving of division problems really ends up being a multiplication problem in reverse.

$5\overline{)30}$ answer 6 $5 \times 6 = 30$

The multiplication table can be used in reverse for division.

For example 30 divided by 5 equals 6:

0	1	2	3	4	5	(6)
1	1	2	3	4	5	5
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

Example:

```
  150 R2
3/ 452
 -3
  15
 -15
   02
```

We first divide the 3 into the four which will go 1 time, and place the 1 above the 4. We place the 3 under the 4 and subtract and place a 1 below. Bring down the 5 to make 15. Then we divide the 3 into the 15, getting 5 placing that after the 1 in the answer and subtracting 15 from 15 leaving 0. Bring down the 2. 3 will not go into 2, so we place a 0 after the 5 in the answer and have two left over (Remainder of 2). This is the old system of Dividing.

Do these Division Problems:

1) $8\overline{)125}$ 2) $7\overline{)8{,}530}$ 3) $3\overline{)3{,}005}$ 4) $9\overline{)7{,}695}$

***TIP: Remember to place the number above the portion being divided at the time and work from left to right from thousand to ones.

Working with larger numbers is done the same way as smaller numbers. We can also use the new concept of thousands, hundreds tens & ones carrying along the zeros to show this.

```
    152 R1
26/ 4,953
   2,600
   1,353
   1,300
      53
      52
       1
```

The only difference to remember is that you are dividing 26 ones into 49 hundreds in the first step.

Second step is 26 ones into 135 tens.

Third step is 26 ones into 53 ones.

Do these Division Problems:

5) 19/ 456 6) 32/ 7,956 7) 27/ 673 8) 21/ 5,384

WHOLE NUMBER
REVIEW QUESTIONS

1) Add

$$\begin{array}{r} 231 \\ 650 \\ 420 \\ 90 \\ +\ 71 \\ \hline \end{array}$$

2) Add Showing Groups of 10's

$$\begin{array}{r} 73 \\ 22 \\ 97 \\ +\ 38 \\ \hline \end{array}$$

3) Subtract

$$\begin{array}{r} 1{,}749 \\ -\ 958 \\ \hline \end{array}$$

4) Multiply

$$\begin{array}{r} 798 \\ \times\ 53 \\ \hline \end{array}$$

5) Divide

$74\overline{)31{,}672}$

FRACTIONS and FRACTIONAL NUMBERS

When we use numerals in a certain way to express a portion of one (1), we call the numerals - fractional numbers.

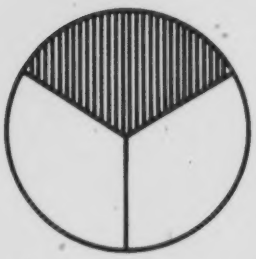
The 1 & 2 used in $\frac{1}{2}$ or 1/2 are fractional numbers

$\frac{1}{2}$ the top number represents how <u>many</u> equal parts we mean.
the lower number always represents how many equal parts the whole thing has.

In this square, the whole thing has two equal parts.

The shaded part is 1 part.

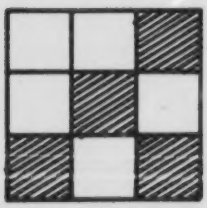
The whole thing has 2 parts so the shaded area is represented by $\frac{1}{2}$ of the whole.



This circle is divided into 3 equal parts.

The shaded part is 1 of 3 equal parts or the shaded area is $\frac{1}{3}$ of the whole.

The unshaded area is $\frac{2}{3}$ of the whole.

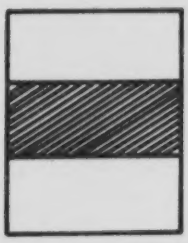


This whole square is divided into 9 equal parts.

We have shaded 4 of the equal parts.

The fraction we shaded is $\frac{4}{9}$ of the whole.

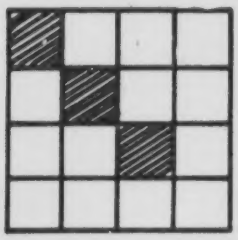
What are the fractional numbers representative for the following shaded parts of the figures.

1) 

Ans.

2)

Ans.

3) 

Ans.

So far, we have used numbers on top of each other with a line drawn between them.

This line is called a fraction bar.

$$\frac{\text{numerator}}{\text{denominator}} \quad \frac{1}{6} \longleftarrow \text{Fraction Bar}$$

In mathematics, this bar - or slash / separating two quantities always means the top quantity is divided by the lower quantity.

$\frac{3}{4}$ or $3/_4$ really means $3 \div 4$ or $4\overline{)3}$

If the numerator and denominator of a fraction are the same, the result is one (1).

$$\frac{4}{4} = 1 \qquad \frac{8}{8} = 1$$

With this same fractional concept, we can represent a whole number in fractions.

$$8 = \frac{8}{1} \qquad 4 = \frac{4}{1}$$

Units of measure may be expressed as fractions.

For example:

Miles per hour = $mi/_{hr}$

Feet per second = $ft/_{sec}$

Million gallons per day = $MG/_D$ or MGD

Gallons per square foot = $gal/_{ft^2}$

Peter Principle

In an organization an individual will rise to his level of incompetence.

Both numerator and denominator in a fraction may be multiplied by the same number without changing the value of the fraction.

$\frac{3}{5} \times 2 = \frac{6}{10}$ $\quad$ $\frac{1}{3} \times 5 = \frac{5}{15}$ $\quad$ $\frac{3}{4} \times 5 = \frac{15}{20}$

same $\quad$ same $\quad$ same

You may also divide both the numerator and denominator by the same number and not change the value of the fraction.

$\frac{9}{18} \div 9 = \frac{1}{2}$ $\quad$ $\frac{9}{12} \div 3 = \frac{3}{4}$ $\quad$ $\frac{10}{25} \div 5 = \frac{2}{5}$

same $\quad$ same $\quad$ same

Dividing the numerator and the denominator by a same number is called "reducing to simplest terms."

When the numerator is greater than a denominator (such as $\frac{9}{4}$) this is called an improper fraction. The following are improper fractions:

$\frac{3}{2}$ $\quad$ $\frac{4}{3}$ $\quad$ $\frac{5}{4}$ $\quad$ $\frac{10}{3}$ $\quad$ $\frac{21}{6}$

An improper fraction may express a whole number plus a fraction. This is called a mixed number.

Fraction $\frac{9}{4}$ $\qquad$ $9 \div 4 = 2\frac{1}{4}$

$$4\overline{)9}\ \ 2\tfrac{1}{4} \quad \text{mixed number}$$
$$\underline{8}$$
$$1 \quad \text{remainder over}$$
$$4 \quad \text{the denominator}$$

Some mixed numbers are:

$2\frac{1}{2}$ $\quad$ $6\frac{7}{8}$ $\quad$ $1\frac{2}{3}$ $\quad$ $16\frac{1}{4}$ $\quad$ $23\frac{5}{6}$

There are times in this process when we get a mixed number and a fraction that can be reduced to <u>simpler terms</u>.

$\frac{15}{6} =$ $6\overline{)15}\ \ 2 + \frac{3}{6}$ or $2\frac{1}{2}$

$$\underline{12}$$
$$\frac{3}{6}$$

$\frac{3}{6} \div 3 = \frac{1}{2}$

Exercise:

4) What whole number does $\frac{6}{6}$ represent? _____

5) What whole number does $\frac{12}{6}$ represent? _____

6) What mixed number does $\frac{26}{5}$ represent? _____

7) What would $\frac{20}{40}$ be when reduced? _____

8) Reduce to simplest terms $\frac{6}{36}$ = _____

9) Change to a mixed number $\frac{27}{10}$ = _____

10) Change to mixed number and reduce $\frac{21}{6}$ = _____

11) What whole number does $\frac{3}{1}$ represent? _____

12) What whole number does $\frac{25}{1}$ represent? _____

Hal says:

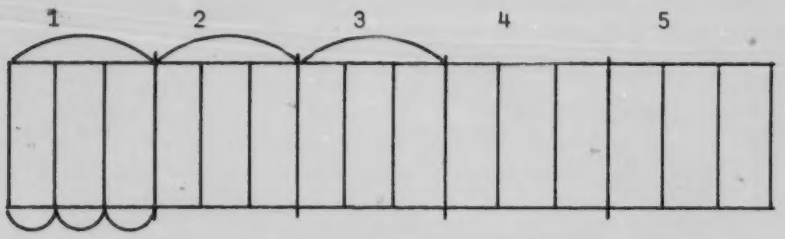
You can tell the age of a boy by the price of his toy.

ADDITION OF FRACTIONS

Like fractions all have the same number below the fraction bar.

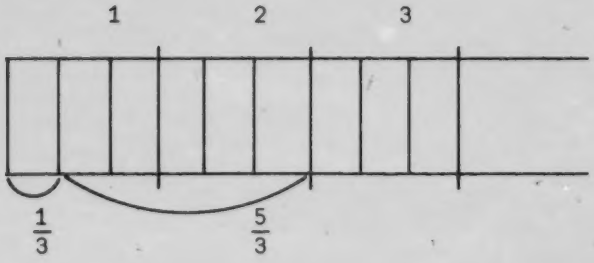
Such as: $\frac{1}{3}$ $\frac{2}{3}$ $\frac{5}{3}$ $\frac{7}{3}$

When we add like fractions we can use the same concept used with whole numbers except each unit is divided into the number below the fraction bar.



$\frac{1}{3}$ $\frac{1}{3}$ $\frac{1}{3}$ etc.

When we add $\frac{1}{3} + \frac{5}{3} = \frac{6}{3}$ or whole number 2



What we do in the case of like fractions is to add the numerators and place the total over the common denominator.

Such as: $\frac{1}{4} + \frac{3}{4} + \frac{6}{4} = \frac{10}{4} =$ $4\overline{)10}$ R2 = 2 2/4 = 2 1/2

$\underline{8}$

2

*Don't forget to reduce to mixed number and reduce to lowest terms.

Add the following Like Fractions:

1) $\frac{1}{8} + \frac{7}{8} + \frac{3}{8} =$

2) $\frac{3}{16} + \frac{2}{16} + \frac{7}{16} =$

3) $\frac{7}{12} + \frac{3}{12} + \frac{1}{12} =$

Unlike fractions are fractions with different numbers in the denominators.

Some unlike fractions are: $\frac{1}{7}$ $\frac{3}{5}$ $\frac{1}{2}$ $\frac{3}{4}$

Suppose we want to add $\frac{1}{2} + \frac{1}{3}$

Since the denominators are not alike we can not use the rule for adding like fractions.

But by using a rule about fractions from 3 previous lessons "You may multiply both the numerator and denominator by the same number and not change the value of the fraction.

Such as: $\frac{1}{2} \times \frac{3}{3} = \frac{3}{6}$ (same value as $\frac{1}{2}$)

Also: $\frac{1}{3} \times \frac{2}{2} = \frac{2}{6}$ (same value as $\frac{1}{3}$)

Now both fractions have same (or common) denominator.

$$\frac{3}{6} + \frac{2}{6}$$

This we can add with our rule:

$$\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$

This process of converting fractions with different denominators to fractions having the same denominator is called finding the Common Denominator.

The easiest way to find the common denominator is to multiply the two denominators.

$$\frac{1}{3} \times \frac{2}{5}$$

3 x 5 = 15 the Common Denominator

To get the numerators we take the old denominator and divide it into new common denominator giving you a number which we multiply times the old numerator to give us the new numerator.

$$\frac{1}{3} = \frac{5 \times 1}{15} = \frac{5}{15}$$

And likewise we change:

$$\frac{2}{5} = \frac{3 \times 2}{15} = \frac{6}{15}$$

For example if we want to add $\frac{1}{5} + \frac{3}{4}$

Multiply 5 x 4 = 20 (common denominator)

Divide $5/\overline{20}^{4}$

Divide $4/\overline{20}^{5}$

Thus

$$\begin{array}{rl} & \frac{1}{5} = \frac{4 \times 1}{20} = \frac{4}{20} \\ + & \frac{3}{4} = \frac{5 \times 3}{20} = \frac{15}{20} \\ \hline & \frac{19}{20} \end{array}$$

We usually just write:

$$\begin{array}{rl} & \frac{4}{20} \\ + & \frac{15}{20} \\ \hline & \frac{19}{20} \end{array}$$

TIP

* So when we look for a Common Denominator, we are looking for a number that the Denominators will Divide into evenly.

This last tip should always be kept in mind when looking for a Common Denominator.

In the case of adding: $\frac{1}{3} + \frac{5}{12}$

Do not multiply 3 x 12 = 36

Common Denominator

12 is the Common Denominator

Both 3 and 12 divide evenly into 12 -- $3\overline{)12}^{4}$ $12\overline{)12}^{1}$

$$\frac{1}{3} = \frac{4 \times 1}{12} = \frac{4}{12}$$

$$+ \frac{5}{12} = \frac{5 \times 1}{12} = \frac{5}{12}$$

$$\frac{9}{12}$$

The Problem can be worked either way but do not work with figures larger than are necessary.

So to keep with smallest Common Denominators, we take the larger denominator and double it and check if smaller denominators will go into that figure - if not we triple it and try dividing.

Such as: $\frac{1}{6} + \frac{1}{8}$

Will 6 go evenly into 8 - No
Will 6 go evenly into 16 - No
Will 6 go evenly into 24 - Yes - This would be the Common Denominator.

What is the common denominator of these pairs of Fractions?

4. $\frac{1}{3}$ and $\frac{3}{4}$ _____

5. $\frac{5}{6}$ and $\frac{3}{4}$ _____

Now add these unlike fractions.

6. $\frac{5}{6} =$
$+ \frac{1}{3} =$
_____

7. $\frac{3}{8} =$
$+ \frac{1}{3} =$
_____

8. $\frac{1}{9} =$
$+ \frac{1}{12} =$
_____

9. $\frac{4}{5} =$
$+ \frac{2}{3} =$
_____

We can add any number of unlike fractions as long as we find a common denominator.

Example:
$$\begin{array}{rcl} \frac{1}{3} & = & \frac{20}{60} \\ \frac{1}{4} & = & \frac{15}{60} \\ \frac{1}{5} & = & \frac{12}{60} \\ \hline & & \frac{47}{60} \end{array}$$

Add these groups of unlike fractions.

10.
$$\begin{array}{r} \frac{1}{3} \\ \frac{1}{8} \\ + \quad \frac{3}{8} \\ \hline \end{array}$$

11.
$$\begin{array}{r} \frac{1}{6} \\ \frac{3}{5} \\ + \quad \frac{1}{10} \\ \hline \end{array}$$

We will now consider adding fractional numbers represented by mixed numerals.

Suppose we want to add $4\frac{1}{7} + 3\frac{3}{7}$.

First change each number to an improper fraction.

$$4\frac{1}{7} = \frac{29}{7} \text{ and } 3\frac{3}{7} = \frac{24}{7}$$

Add the improper fractions:
$$\begin{array}{r} \frac{29}{7} \\ + \quad \frac{24}{7} \\ \hline \frac{53}{7} \end{array}$$

Change the answer to a mixed number $7\frac{4}{7}$

This is a lot of work for a simple addition problem when we know that $4\frac{1}{7}$ really means $4 + \frac{1}{7}$ and $3\frac{3}{7}$ means $3 + \frac{3}{7}$

If we add the whole numbers first $4 + 3 = 7$, then add fractions $\frac{1}{7} + \frac{3}{7} = \frac{4}{7}$

The answer is $7 + \frac{4}{7}$ or $7\frac{4}{7}$

So the rule for adding mixed numbers is to add the fractions separate from the whole number and then add the two answers.

There are times when the fractions will total a mixed number. So we must add the whole number in with the whole numbers.

Example:

$$\begin{array}{r} 13\frac{4}{9} = \frac{8}{18} \\ 73\frac{5}{6} = \frac{15}{18} \\ \hline 86 + \frac{23}{18} \text{ or } 1\frac{5}{18} \end{array} \qquad \begin{array}{r} = 86 \\ + 1\frac{5}{18} \\ \hline 87\frac{5}{18} \end{array}$$

Do these Exercises:

12. $79\frac{7}{12} + 13\frac{7}{9}$

13. $45\frac{3}{8} + 17\frac{15}{16}$

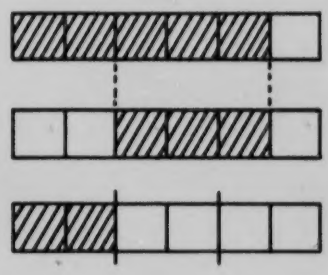
SUBTRACTION OF FRACTIONS

When subtracting like fractions we really subtract the numerators and keep the denominator which is common.

Let us subtract $\frac{3}{6}$ from $\frac{5}{6}$

We have a chart

The shaded area represents $\frac{5}{6}$



If we take away $\frac{3}{6}$

We would have $\frac{2}{6}$ or $\frac{1}{3}$

or

Another Example: $\frac{11}{18} - \frac{5}{18} = \frac{6}{18}$ Lowest terms $\frac{1}{3}$

Subtract the following examples:

1. $\frac{13}{16} - \frac{3}{16} =$ _____

2. $\frac{10}{13} - \frac{7}{13} =$ _____

3. $\frac{41}{50} - \frac{3}{50} =$ _____

When subtracting with unlike fractions, we follow the same rule as adding unlike fractions. We must first find the common denominator and express the fractions in relation to this common denominator.

Example: $\frac{5}{6} - \frac{4}{9}$

common denom. $\frac{15}{18} - \frac{8}{18}$ Now we subtact the numerators and keep the common denominator giving us $\frac{7}{18}$

The most common way to write subtraction of fractions is as follows:

$$\begin{array}{r} \frac{5}{6} = \frac{15}{18} \\ -\ \frac{4}{9} = \frac{8}{18} \\ \hline \frac{7}{18} \end{array}$$

Subtract the following:

4. $\begin{array}{r} \frac{3}{4} \\ -\ \frac{1}{16} \\ \hline \end{array}$

5. $\begin{array}{r} \frac{7}{8} \\ -\ \frac{1}{6} \\ \hline \end{array}$

When subtracting mixed numerals we subtract using the same rules as we did when adding mixed numerals. First convert the fractions to fractions with a common denominator, subtract the fractions, subtract the whole numbers, and combine the whole number and fractions for the answer.

Example: $26\ \frac{13}{20}$ or $26 + \frac{13}{20}$

$-\ 12\ \frac{3}{5}$ $\quad -\ 12 + \frac{12}{20}$

$$\begin{array}{rcl} 26 & + & \frac{13}{20} \\ -\ 12 & + & \frac{12}{20} \\ \hline 14 & + & \frac{1}{20} \end{array} \quad = 14\ \frac{1}{20}$$

Do the following subtraction:

6. $5\frac{3}{5} - 2\frac{1}{4}$

7. $53\frac{7}{12} - 12\frac{3}{8}$

When we subtract Mixed Numerals we sometimes have a problem much like subtracting whole numbers.

If we try to subtract the following: $7\frac{1}{6} - 2\frac{5}{6}$

$6\not{7}\frac{1}{6} + \frac{6}{6} = \frac{7}{6}$

$-2\frac{5}{6} \longrightarrow \frac{5}{6}$

$4 \qquad \frac{2}{6}$

In the fractional portion of the problem we have a common denominator, but we can't take 5 from 1 in the numerator, so we borrow a whole 1 from the 7 leaving 6.

Then we express the borrowed 1 as $\frac{6}{6}$ and add that to the $\frac{1}{6}$ giving us $\frac{7}{6}$.

or $4\frac{1}{3}$

Now we can subtract $\frac{5}{6}$ from $\frac{7}{6} = \frac{2}{6}$ or $\frac{1}{3}$.

Subtract 2 from 6 = 4

Adding 4 and $\frac{1}{3}$ giving a total of $4\frac{1}{3}$.

Now with this principle of borrowing a whole one and expressing in a fraction, do the following subtractions.

8. $9\frac{2}{7} - \frac{3}{7}$

9. $9\frac{3}{8} - 4\frac{15}{16}$

10. $25\frac{3}{5} - 14\frac{2}{3}$

MULTIPLICATION OF FRACTIONS

When one of the factors is a whole number, such as $\frac{2}{3} \times 2$, we multiply the whole number by the numerator of the fraction and put the product over the denominator.

Example: $\frac{2}{3} \times 2 = \frac{4}{3}$

Then we convert this to a mixed number by dividing the denominator into the numerator.

Example: $3\overline{)4}$ $\quad 1\frac{1}{3}$ Mixed Number.

3

$\frac{1}{3}$

To use a graphic picture of this procedure:

$\frac{2}{3} \times 2 = \frac{4}{3}$ or $1\frac{1}{3}$

Now you should be able to do these exercises. Don't forget to express the product in the simplest form.

1. $3 \times \frac{3}{11} =$

2. $5 \times \frac{3}{7} =$

3. $5 \times \frac{1}{10} =$

Multiplication when both numerals are expressed in fractions.

The rule, in this case, is to multiply the numerators, then multiply the denominators, giving you a new numerator and denominator.

Example: $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$

Sometimes the product can be reduced to simpler terms. When it can be reduced it should be reduced.

Example: $\frac{2}{5} \times \frac{3}{4} = \frac{6}{20} = \frac{3}{10}$

Now with this principle known we can do the following multiplications of fractions:

4. $\frac{6}{5} \times \frac{3}{5} =$

5. $\frac{1}{3} \times \frac{6}{9} =$

6. $\frac{8}{9}$
$\times \frac{5}{7}$

Next we will multiply a mixed number by a whole number.

The rule for this is to first multiply the whole number by the whole number part of the mixed number. Then we multiply the whole number by the numerator of the fractional part of the mixed number, then add the two.

Example:

$$9 \times 4\frac{1}{6} = 9 \times 4 + 9 \times \frac{1}{6} = 36 + \frac{9}{6} = 36 + 1\frac{3}{6}$$

$$= 36 + 1\frac{1}{2} = 37\frac{1}{2}$$

or

$$\begin{array}{r} 4\frac{1}{6} \\ \times\ 9 \\ \hline \end{array} = \begin{array}{r} 4 + \frac{1}{6} \\ \times\ 9 \\ \hline 36 + \frac{9}{6} \end{array} = 36 + 1\frac{1}{2} = 37\frac{1}{2}$$

With the above example you can see we have two multiplications and one addition to do to solve the problem. If we were to multiply two mixed numbers we would have to do the process twice which makes six operations to solve the problem. Large numbers would make the procedure very cumbersome.

If we were to change all numbers into fractions (proper or improper) and multiply the fractions, the procedure would be much simpler.

Example: $1\frac{2}{3} \times 4\frac{1}{5}$

$$\frac{5}{3} \times \frac{21}{15} = \frac{105}{45} \div 15 = \frac{7}{1} = 7$$

Example: $5 \times 2\frac{1}{5}$

$$\frac{5}{1} \times \frac{11}{5} = \frac{55}{5} \div 5 = \frac{11}{1} = 11$$

There is only one remaining problem with this new system and that is the simplifying at the end.

There is a way to lessen this problem by what we call cross division or <u>canceling out</u>.

The rule is that <u>with the multiplication of fractions</u> we can divide the numerator of one fraction and the denominator of the other fraction with a number and not change the value of the fractions as a whole. We can simplify the factors of a fraction in the same way:

We divided both numerator and denominator by 5.

$$\frac{1}{\cancel{5}_{1}} \times \frac{\overset{2}{\cancel{10}}}{3} = \frac{1}{1} \times \frac{2}{3} = \frac{2}{3}$$

We divide the 3 and the 21 by 3 and the 5 and the 5 by 5.

$$\frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{3}}} \times \frac{\overset{7}{\cancel{21}}}{\underset{1}{\cancel{5}}} = \frac{1}{1} \times \frac{7}{1} = \frac{7}{1} \text{ or } 7$$

We divide the 5 and the 10 by 5 and the 2 and the 4 by 2.

$$\frac{\overset{1}{\cancel{5}}}{\underset{2}{\cancel{10}}} \times \frac{\overset{1}{\cancel{2}}}{\underset{2}{\cancel{4}}} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

The process is to change any whole numbers into fractions by putting the number over 1. Then change any mixed numbers to improper fractions. Next, use cross division or canceling out and then multiply the numerators and multiply denominators. Last, if necessary, simplify to lowest terms or change to a mixed number.

Example: $2\frac{2}{3} \times 6\frac{1}{4} = \frac{\overset{2}{\cancel{8}}}{3} \times \frac{25}{\cancel{4}_{1}} = \frac{50}{3} = 16\frac{2}{3}$

Following these rules do these exercises:

7. $3\frac{3}{4} \times 1\frac{3}{5} =$ _____

8. $\frac{2}{3} \times 3\frac{1}{5} =$ _____

9. $1\frac{2}{9}$
$\times \frac{3}{22}$

_____

DIVISION OF FRACTIONS

In a division problem with fractions, we have a dividend which will be divided by a divisor to give you a quotient.

Example: $\frac{1}{2} \div \frac{4}{5} = \frac{\frac{1}{2}}{\frac{4}{5}}$

To divide $\frac{1}{2}$ by $\frac{4}{5}$ we first find the reciprocal of the divisor $\frac{4}{5}$.

The reciprocal of any fraction is the fraction turned upside down or inverted.

Example: $\frac{4}{5}$ has a reciprocal of $\frac{5}{4}$

To divide $\frac{1}{2}$ by $\frac{4}{5}$ we simply invert $\frac{4}{5}$ (turn it upside down) and multiply as usual.

$$\frac{1}{2} \div \frac{4}{5} \left(\frac{\frac{1}{2}}{\frac{4}{5}} \right) \quad \frac{1}{2} \times \frac{5}{4} = \frac{5}{8}$$

Remember we follow all the rules of multiplying fractions when we do this.

Example: $\frac{2}{3} \div \frac{3}{4}$ Invert and multiply $\frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$

Example: $\frac{9}{16} \div \frac{15}{8}$ Invert and multiply $\frac{\cancel{3}}{\cancel{16}_{2}} \times \frac{\cancel{8}^{1}}{\cancel{15}_{5}} = \frac{3}{10}$ (use cancelling)

Example: $1\frac{13}{15} \div 6\frac{2}{5}$

$\frac{28}{15} \div \frac{32}{5}$ Change to improper fractions.

$\frac{\cancel{28}^{7}}{\cancel{15}_{3}} \times \frac{\cancel{8}^{1}}{\cancel{32}_{8}} = \frac{7}{24}$ Invert the divisor and cancel out.

Now we can do these exercises:

1. $\frac{3}{11} \div \frac{2}{3} =$ _____

2. $2\frac{1}{10} \div 3\frac{1}{2} =$ _____

3. $\frac{4}{5} \div 3\frac{1}{2} =$ _____

Mary Principle

If many individuals remain too long at their level of incompetence, they will destroy the organization because their presence demonstrates to others that competence is not a prerequisite for success.

FRACTION REVIEW QUESTIONS

Change these improper fractions to Mixed Numbers:

1. $\frac{13}{8} =$ 2. $\frac{25}{7} =$ 3. $\frac{14}{4} =$ 4. $\frac{7}{3} =$

Reduce these fractions to lowest terms:

5. $\frac{15}{45} =$ 6. $\frac{7}{21} =$ 7. $\frac{216}{324} =$ 8. $\frac{176}{42} =$

Add:

9. $\frac{1}{2} + \frac{1}{5} + \frac{3}{5}$ 10. $\frac{3}{12} + \frac{7}{10} + \frac{9}{5}$ 11. $\frac{1}{6} + \frac{3}{12} + \frac{9}{32}$

Subtract:

12. $\frac{2}{3} - \frac{4}{9}$ 13. $\frac{3}{5} - \frac{7}{12}$ 14. $\frac{1}{12} - \frac{1}{25}$

Multiply:

15. $\frac{1}{5} \times \frac{3}{7} =$ 16. $\frac{7}{10} \times \frac{6}{2} =$ 17. $\frac{5}{7} \times \frac{3}{4} =$

Divide:

18. $\frac{1}{7} \div \frac{3}{5} =$ 19. $\frac{1}{6} \div \frac{7}{10} =$ 20. $3 \div \frac{7}{5} =$

DECIMALS

In our number system the regular whole numbers are grouped in ones, tens, hundreds, thousands, and so on, increasing in multiples of 10 as we move left from the decimal point.

Example:

1. = ones
10. = tens
100. = hundreds
1000. = thousands
etc.

The fractions of numbers are grouped to the right of the decimal point and decrease in multiples of tenths as we move to the right.

Example:

$.1 = \frac{1}{10}$ = tenths
$.01 = \frac{1}{100}$ = hundredths
$.001 = \frac{1}{1000}$ = thousandths
etc.

The numbers to the left of the decimal point are the whole numbers and those to the right of the decimal point are the fractional parts.

Example:

2.5
two and five tenths
$2 + \frac{5}{10}$

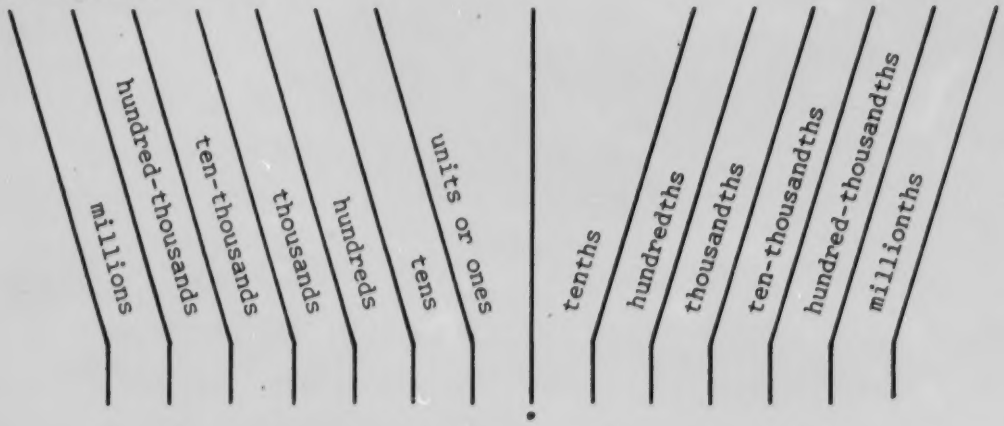
Example:

16.34
sixteen and thirty four hundredths
$16 + \frac{34}{100}$

Example:

321.296
three hundred twenty one and two hundred ninety six thousandths
$321 + \frac{296}{1000}$

The following chart names the six places to the left and to the right of the decimal point.



ADDITION OF DECIMALS

When we add whole numbers we put all the ones in a column, all the tens in the next to the left, and so on. Then it is easy to add all the ones, tens, etc.

Example: Add - 5 + 216 + 23 + 642 =

```
   5
+ 216
+  23
+ 642
-----
 886
```

In decimals we do the same thing with all the tenths, hundredths, thousandths, etc.

Example: Add - 0.3 + .0592 + .167 + .203 =

```
   .3
+  .0592
+  .167
+  .203
-------
  .7292
```

This will give you all the decimal points in a column no matter how many numbers we have before or after the decimal points.

Example:
```
03.5600
10.7320
01.7000
03.6784
```

To make the addition even easier we add zeros before whole numbers and after decimal numbers to square up the problem. These do not change the value of the numbers.

Then we add in the usual way but remembering to include the decimal point in the answer in the same column as the other decimal points.

Example:
```
02.859
25.980
04.790
+ 05.400
--------
 39.029
```

Now we can do the following exercises:

1. 13.6 + 9.7 + 3.5 + 10.4 =

ans. _____

2. 9.5 + .016 + 32.3 + 7.0312 =

ans. _____

3.
$$\begin{array}{r} 0.3 \\ 4.0 \\ 0.7 \\ 5.0 \\ 0.6 \\ +\ 8.0 \\ \hline \end{array}$$

ans. _____

SUBTRACTION OF DECIMALS

The same rules we used for addition are used in subtraction so long as you keep the columns in line and the decimals in line.

Then we subtract in the usual manner.

Example:
$$\begin{array}{r} 6.57 \\ -\ 2.34 \\ \hline 4.23 \end{array}$$

The same rules of borrowing for subtraction of whole numbers will still apply in decimal subtraction.

Example:
$$\begin{array}{r} {\scriptstyle 3\ \ 2} \\ 4.\cancel{3}65 \\ -\ 2.780 \\ \hline 1.585 \end{array}$$

Now we can do these exercises:

4. $\begin{array}{r} 4.9 \\ -\ 3.0 \\ \hline \end{array}$

ans. _____

5. $\begin{array}{r} 39.1 \\ -\ 5.431 \\ \hline \end{array}$

ans. _____

6. 17.369 - 4.29 =

ans. _____

MULTIPLICATION OF DECIMALS

In multiplication with whole numbers we went over the principle of the powers of 10. Any number multiplied by ten would be that number plus a 0 on the right end.

Example:
$$\begin{array}{r} 35 \\ \times\ 10 \\ \hline 350 \end{array}$$

Every whole number has an unwritten decimal point to the right. Thus, when we multiply by ten we are really moving the decimal point one place to the right.

Example with decimals:
$$\begin{array}{r} 35. \\ \times\ 10 \\ \hline 350. \end{array}$$

This same principle still applies no matter where the decimal point is located.

Example: 32.6 x 10 = 326.

Example: 326.597 x 10 = 3265.97

Example: 0.0621 x 10 = 0.621

When we multiply by 100 we move the decimal point two places to the right.

Example: 3.7632 x 100 = 376.32

Example: .01623 x 100 = 1.623

This goes on through the millions if necessary.

Now do these exercises:

7. $\begin{array}{r} 43 \\ \times\ 10 \\ \hline \end{array}$

8. $\begin{array}{r} .35 \\ \times\ 10 \\ \hline \end{array}$

9. $\begin{array}{r} 356.1 \\ \times\ 100 \\ \hline \end{array}$

Now we will multiply decimal numbers by decimal numbers. In this we do not have to keep the decimals in a column. We proceed to multiply just as we do with whole numbers and put in the decimal point (in the answer) later.

Example:
$$\begin{array}{r} 1.17 \\ \times \quad 2.4 \\ \hline 468 \\ 234 \\ \hline 2808 \end{array}$$

Now to find where the decimal point should be. There are two places to the right of the decimal point in 1.17 ie .17 or .$\underline{x}$ $\underline{x}$ and one place to the right of the decimal point in 2.4 ie .4 or .$\underline{x}$. The sum of these places 2 (.$\underline{x}$ $\underline{x}$) + 1 (.$\underline{x}$) is three and we must insert the decimal point three places to the left of the last digit.

2808 = 2.808

Example:
$$\begin{array}{r} 16.3 \\ \times \; 2.104 \\ \hline 652 \\ 1630 \\ 326 \\ \hline 342952 \end{array}$$

.$\underline{x}$ 1 decimal place
\+ .$\underline{x}$ $\underline{x}$ $\underline{x}$ 3 decimal places

.$\underline{x}$ $\underline{x}$ $\underline{x}$ $\underline{x}$ 4 decimal places

342952. = 34.2952

Now we can do these exercises:

10. $\begin{array}{r} .809 \\ \times \; 9.15 \end{array}$ ans._____

11. $\begin{array}{r} 7.3 \\ \times \quad .9 \end{array}$ ans._____

12. $\begin{array}{r} 7.71 \\ \times \quad 3.9 \end{array}$ ans._____

DIVISION WITH DECIMALS

When we divide any number by 10, 100, 1,000, 10,000 etc., we use the tens principle as in multiplication except we move the decimal to the left.

Example: 341 ÷ 10 = 34.1

341 ÷ 100 = 3.41

341 ÷ 1000 = .341

3.41 ÷ 10 = .341

.0341 ÷ 100 = .000341

With this principle we can now do these exercises:

13. 64.3 ÷ 100 =

14. .146 ÷ 10 =

15. 4 ÷ 1,000 =

16. 465.1 ÷ 100 =

Now we will do Division with Decimals when the divisor is a whole number.

This is done the same as all whole numbers except we have to be sure to put the decimal in the answer.

Example:

```
                 7.19      Quotient
Divisor  9/ 64.71          Dividend
           63
           17
            9
           81
           81
            0
```

Put the decimal in the quotient directly above the decimal in the dividend before doing the numerical division.

Example: $9\overline{)64.71}$

With this we can do these Exercises:

17. $7\overline{)34.86}$ 18. $21\overline{)640.5}$ 19. $45\overline{)188.1}$

Now for Division when both the divisor and dividend are expressed in decimals.

Example: $\frac{2.10}{0.06}$ or $.06\overline{)2.10}$

The procedure here is to move the decimal to the right in the divisor to make a whole number. At the same time we move the decimal in the dividend the same number of points to the right.

.06 → 2.10 →

Giving us $6\overline{)210}$ then divide. This does not change the problem at all but it does make it easier to solve.

Reasoning:

$.06\overline{)2.10}$ can be expressed as a fraction. $\frac{2.10}{.06}$

You can multiply both the numerator and denominator by 100 without changing the fraction.

$\frac{2.10 \times 100}{.06 \times 100}$ Giving us $\frac{210}{6}$ which is $6\overline{)210}$

With this multiplication by tens we can solve a decimal division problem the same way we did when the divisor was a whole number.

Just remember to multiply both the divisor and dividend by the same power of ten.

Example of moving decimal. (or multiply)

$3.569.\overline{)534.769.5}$

giving us: $3569.\overline{)534769.5}$

Now we are ready to do these exercises:

20. $.12\overline{)47.64}$ 21. $.28\overline{).8876}$ 22. $3.7\overline{)1494.8}$

Next we should be able to change decimals to fractions.

A fraction is a division problem.

$\frac{1}{3}$ means $3\overline{)1.000}$

one divided by three.

If you complete the division of any fraction you will get the decimal equivalent of that fraction.

Example: $\frac{2}{5}$ is $5\overline{)2.0}$ $\quad .4$

Example: $\frac{15}{20}$ is $20\overline{)15.00}$ $\quad .75$

$\quad\quad\quad

PERCENTAGE

Percent uses a special symbol "%."

Percent comes from the Latin meaning "per 100."

In other words we are comparing the number to 100.

20% compares 20 to 100.

This comparison is called a ratio and we usually represent a ratio in the form of a fraction:

$$20\% = \frac{20}{100}$$

So 10% is the same as $\frac{10}{100}$

What are the Fractional equivalents of the following?

1. 35% =

2. 15% =

3. 71% =

Per cent can also be expressed as a decimal.

Example: $34\% = \frac{34}{100} = .34$

All three of these are the same thing expressed in a different way.
Four examples of this concept follow:

Percent	Fraction	Decimal
15%	$\frac{15}{100}$	.15
25%	$\frac{25}{100}$	.25
20%	$\frac{20}{100}$	.20
69%	$\frac{69}{100}$	.69

Some of these fractional expressions can be reduced to simpler terms as we did in the fraction section. This does not change the value of the fraction.

Examples: $\frac{20}{100} = \frac{2}{10} = \frac{1}{5}$

$\frac{30}{100} = \frac{3}{10}$

$\frac{25}{100} = \frac{1}{4}$

Change these percentiles to fractions and reduce to simplest terms.

4. 50% =

5. 10% =

6. 80% =

There are times when we have a percentage that is larger than 100. When you change that to a fraction you will get an improper fraction.

Example: $130\% = \frac{130}{100}$

This fraction should be reduced to $1\ \frac{30}{100}$

or further to $1\ \frac{3}{10}$

This reduced fraction expressed as a decimal will be $1\ \frac{3}{10} = 1.3$

Change these percentages to fractions, reduce and express as a decimal.

7. 131% =

8. 245% =

9. 161% =

If you want to convert a percentile directly to a decimal the procedure is very simple. You take the percentile and move the decimal two places to the left. (this is dividing by 100) from previous lessons.

Example: 21% = .21

261% = 2.61

If you want to convert a decimal to a percentile you merely do the reverse. We move the decimal two points to the right. (This is multiplying by 100)

Examples: 4.65 = 465%

.05 = 5%

Now we can change % to decimal and decimal to %.

Convert these examples.

10. 21% =

11. 7.9 =

12. .35 =

13. .23% =

Express in Fractions in their Simplest Form.

14. 45% =

15. 110% =

16. .15 =

17. .07 =

Express in Decimals:

18. 43% =

19. 3.5% =

20. $\frac{1}{5}$ =

21. $\frac{36}{100}$ =

Express in Percentages

22. .157 =

23. 3.8 =

24. $\frac{3}{25}$ =

25. $\frac{9}{100}$ =

The question always comes up as to why we should be able to do these conversions. How is all this manipulation going to be used in solving problems?

When we take a percent of a number, we don't actually multiply by a ratio or a percentage. We first must convert the percentile to a fraction or a decimal fraction. Then it is possible to do the multiplication.

Example: #1

What is 50% of 60 shear pins? Note: "of" means multiply.

50% x 60 = ?

Convert to --- .50 x 60 = 30
or Convert to --- $\frac{1}{2}$ x 60 = 30

Example: #2

If we had 35 shear pins and used 5 shear pins for replacement, what % of shear pins did we use?

The fraction we come up with is $\frac{5}{35}$. Or we are comparing 5 shear pins to 35 shear pins.

$\frac{5}{35} = \frac{1}{7}$ of the shear pins

$\frac{1}{7}$ converted to decimal is $.14285^{+}$

(do the division $\frac{1.00000}{7} = .14285$)

Decimal converted to % = .14285 x 100 = 14.285%

Practical Problems of Percent

26. What is 5% of $60.00?

ans._____

27. What is 20% of 320 days?

ans._____

RATIO AND PROPORTION

Probably the most important proportion used in Sewage treatment is:

$$1 \text{ ppm} = 1 \frac{\text{mg}}{\text{l}} = 8.34 \frac{\text{lbs}}{\text{MG}}$$

As we learned in previous lessons; a fraction is a ratio. We are comparing two numbers.

Example: $\frac{2}{5}$ There are two portions
There are five portions in the whole unit.

The whole is divided into 5 parts and we are representing 2 of those five parts with the fraction $\frac{2}{5}$

A proportion is a statement that two ratios (or fractions) are equal.

Example: 2:5::4:10

This is expressed as: Two is to five as four is to ten.
When we write this as fractions we have:

$$\frac{2}{5} = \frac{4}{10}$$

If we cross multiply this we have proven the proportion (or proven the fractions are equal).

$$\frac{2}{5} \times \frac{4}{10}$$

$$20 = 20$$

With this concept of proportion, we are now ready to put the principal to a practical use.

If we can set up a proportion with one of the numbers missing (or unknown) we can then solve by cross multiplication.

Example: #1

If twelve bolts cost you $1.58, what will six bolts cost?

Set up as a proportion ------- 12:1.58::6:?

Change to a fraction $\frac{12}{1.58} = \frac{6}{?}$

Cross multiply $12 \times ? = 6 \times 1.58$

Divide both sides by 12 $\frac{12 \times ?}{12} = \frac{6 \times 1.58}{12}$

Solve for ? $? = \frac{1.58}{2}$

$? = \$.79$

(six bolts) $? = 79¢$

This example is very simple because we know 6 is just $\frac{1}{2}$ of 12 so we would take $\frac{1}{2}$ of 1.58 and have the answer. But if we were figuring in other numbers it could be more difficult.

Example: #2

We know 12 bolts would cost $1.58. How much would 126 bolts cost?

12:1.58::126:x (use x as the unknown)

Change to a fraction $\frac{12}{1.58} = \frac{126}{x}$

Cross multiply $12x = 126 \times 1.58$

Reduce both sides $\frac{12x}{12} = \frac{126 \times 1.58}{12}$

Solve for x $x = \frac{21 \times 1.58}{2}$

$x = 21 \times 0.79$

$x = \$16.59$

Now we can set up proportions and solve for the unknown.

Do the following solving for "x"

1. 81 : x :: 45 : 10

2. x : 7 :: 350 : 50

3. We have $30.00 to spend on polymers and they cost $1.20 for 10 lbs. How many lbs. of polymer can we buy?

Paul Principle

Individuals in an organization who once functioned at their level of competence often become incompetent as they become uneducated for that level

LINES and DISTANCES

Man has always been interested in questions such as "How Much?," "How Far?," "How Long?," and "How Big?". All of these questions are answered by measuring the distances involved.

We are going to measure the length of lines. These lines may be straight or curved. They may be up, down, across, over, under, through, or around. These lines are called by different names; some of which are length, width, depth, perimeter, radius, diameter, and circumference.

Sometimes it is not possible to determine a distance by direct measurement, such as across water, and the distance must be determined mathematically.

In order to be accurate in our measurement and, at the same time, be consistant, we must use a standard system.

There are two standard systems of measure used in waste water treatment. They are the English system and the Metric system. The English system is widely used and is the one with which we are all familiar. The metric system has always been used in the laboratory and is now being gradually adopted as the world standard of measure.

Tables of the English system, the Metric system, and the conversion factors to change from the English to the Metric system are included on pages 177-179.

Two commonly used comparisons of measure are the inch in the English system and the centimeter in the Metric system. Comparing the two standards the inch is approximately $2\frac{1}{2}$ times as long as the centimeter.

To do the measuring in this exercise, each student should have a ruler with both the English and Metric unit of measure. This will be two measuring instruments in one.

The twelve inch ruler will have about thirty and one half centimeters.

The standard Ruler is usually marked off in inches and then sub-divided into $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$ markings.

Example:

You will notice the inch is the long line.
The half inch the next smaller.
The quarter inch the next smaller.
The eighth inch the next smaller.
The sixteenth inch the smallest.

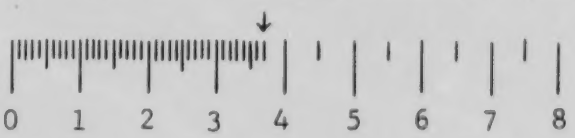
This is done to facilitate reading of the ruler.

The reason we sub-divide each of these scales is to make it easier to read.

Inches are divided into fractions such as $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, and $\frac{1}{16}$ etc. In lengths over an inch, we express our measures in whole numbers and fractions such as one and one-half ($1\frac{1}{2}$) inches etc.

In the case of the metric system, the centimeter is divided into 10 millimeters. We can express our measure in centimeters plus millimeters, or translate it all into millimeters. The most common way to express this measure is by using decimal fractions such as:

3.7 centimeters which means seven millimeters beyond three centimeters.



Measure the following line segments:

Using Inches

1. A B

2. A B

3. A B

Using Centimeters

4.

5.

6.

ANSWER SHEET

1. 2 inches
2. $2\frac{3}{4}$ inches
3. $4\frac{1}{8}$ inches
4. 9 cm or 90 mm
5. 7.9 cm or 79 mm
6. 11.5 cm or 115 mm

Weiler's Law

Nothing is impossible for the man who doesn't have time to do it himself

PERIMETERS & AREAS

Linear distances are important by themselves. We need to know many different distances. For example: How much sewer pipe do we need? How long is the grit chamber? How much fencing do we need to surround and protect our plant? How much chain do we need to operate our scrapers?

We are not only interested in the length of a treatment unit but also in the distances around it. The distance around a triangular or rectangular unit is called the perimeter. The perimeter or distance around a circular unit has a special name and is called the circumference. We need to be able to measure or calculate the length of a straight, rectangular or circular weir.

Remember distances are measured in linear units i.e. inches (in), feet (ft), centimeters (cm), meters (m), etc.

We also must be concerned with the measurement of surface areas. For the purpose of processing we will limit this discussion to areas of flat surfaces. We need to know: How much surface do we have for settling? How much surface do we have on our trickling filter? Are our drying beds large enough?

Areas are measured in square units i.e. square inches (in^2), square centimeters (cm^2), square feet (ft^2).*

*Incidentally this little superscript or exponent ^{2} means to multiply by itself.

For example: $ft^2 = ft \times ft = feet \times feet$

$r^2 = r \times r = radius \times radius$

$D^2 = D \times D = Diameter \times Diameter$

The surface of the structures that we will study are in one plane or in other words have a flat surface.

The distance around this surface is called perimeter or, in the case of a circle, it is called the circumference.

The number of sides of a geometric figure may be used as a general means of classification.

The following is a list of this general classification.

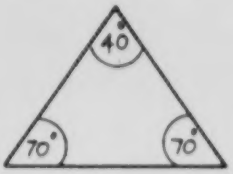
Name of Figure	Number of Sides
Triangle	3
Quadrilateral	4
Pentagon	5
Hexagon	6
Septagon	7
Octagon	8
Nonagon	9
Decagon	10
Circle	No Sides

The Triangle, Quadralateral, and Circle are the three general classifications with which we are going to work.

TRIANGLE

General- All triangles have three sides and three angles. The size of the inside angles always adds up to 180°.

Examples:

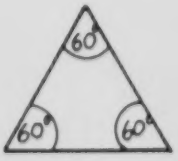


Under this general heading we have triangles which have special qualifications. These are given descriptive names.

Equilateral Triangles

Special qualifications: All three sides are equal and all three angles are each 60° and equal.

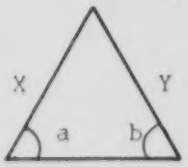
Example:



Isosceles Triangle

Special qualifications: The two sides are equal and the opposite angles are equal.

Example:



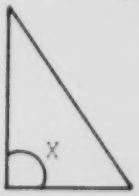
X & Y are equal

a & b are equal

Right Triangle

Special Qualifications: One of the angles is a 90° or right angle.

Example:

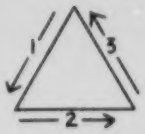


X is 90° or a right angle.

We also give a name for the triangle that doesn't meet any special qualifications. This is a Scalene Triangle with none of the sides or angles equal.

Perimeter of Triangles

The perimeter of a triangle is the distance around the figure or the sum of the lengths of the three sides.



Formula: Perimeter = Side 1 + Side 2 + Side 3.

$P = S_1 + S_2 + S_3$

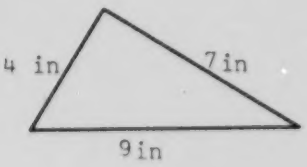
This formula applies to all triangles.

Example:

$P = S_1 + S_2 + S_3$

$P = 4 \text{ in} + 7 \text{ in} + 9 \text{ in}$

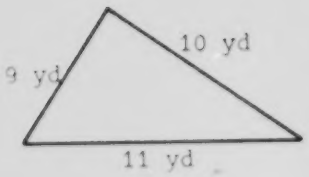
$P = 20 \text{ in}$



Problems:

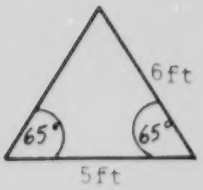
Find the perimeter of these triangles:

1.



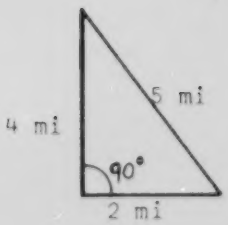
ans._____

2.



ans._____

3.



ans._____

Area of a Triangle

We also need to know the area of a triangle. This area is the number of square units contained within the triangle.

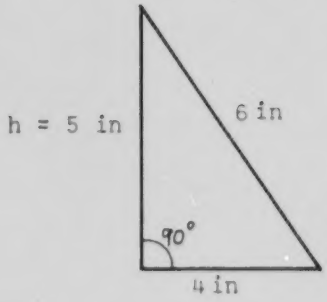
We do not have a simple way to measure area directly. Using dimensions which we can measure and a formula (formulas are directions or instructions) we can calculate the area.

The area of a triangle is the base times the height divided by 2.

Formula: $\text{Area} = \frac{\text{base} \times \text{height}}{2}$

$$A = \frac{b \times h}{2}$$

Example:



$$A = \frac{b \times h}{2}$$

$$A = \frac{4 \text{ in} \times 5 \text{ in}}{2}$$

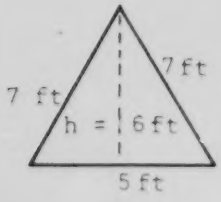
$$A = \frac{20 \text{ in}^2}{2}$$

$$A = 10 \text{ in}^2$$

Problems:

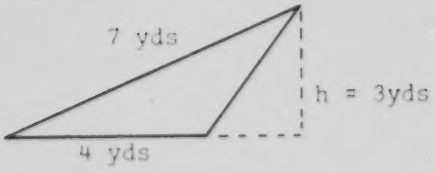
Find the area of tnese triangles.

4.



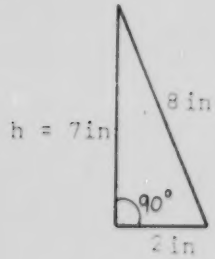
ans._____

5.



ans._____

6.



ans._____

QUADRILATERALS

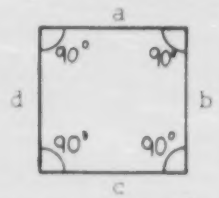
A quadrilateral is any figure that has four sides and four angles. The inside angles add up to 360°.

If there is a relationship between some of the angles and sides we give these figures special names.

There are three of these figures called square, rectangle, and trapezoid which we will use in our work.

A square is a quadrilateral in which; all four sides are equal and all four angles are 90°.

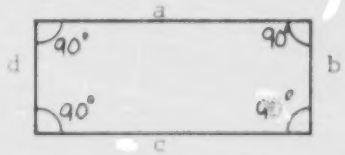
Example:



all angles are 90° and equal
all sides (a, b, c, d) are equal.

The rectangle is a quadrilateral in which; all four angles are 90° and the opposite sides are parallel and equal in length.

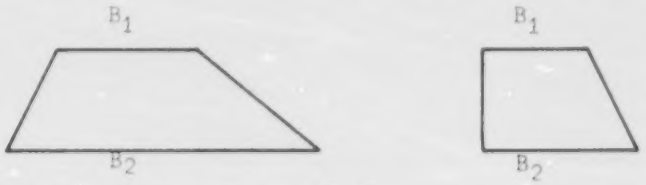
Example:



all angles are 90° and equal
side a + c are equal
side b + d are equal
but side a is not equal to b or d.

The trapezoid is a quadrilateral in which the two bases are parallel. No sides are necessarily equal and no angles are necessarily equal.

Example:



The two bases B_1 and B_2 are parallel.

Like any plane figure, we should be able to find out two things about these figures. They are the perimeter or distance around, and the area which is the square units enclosed by the sides.

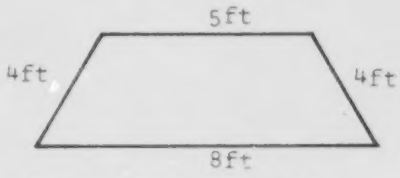
The perimeter of a Quadrilateral

The perimeter of a quadrilateral is the total length of all four sides.

The formula for this is $P = S_1 + S_2 + S_3 + S_4$

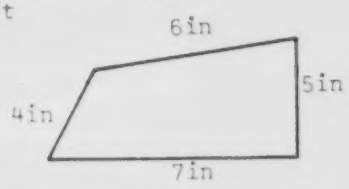
This formula works on all types of quadrilaterals.

Example 1:



P = 5ft + 8ft + 4ft + 4ft
P = 21ft

Example 2:



P = 4in + 6in + 7in + 5in
P = 22in

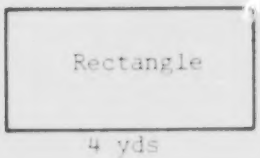
Problems:

Find the perimeter of these quadrilaterals.

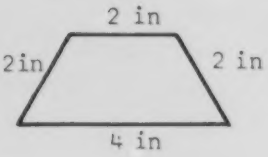
7.

ans. _____

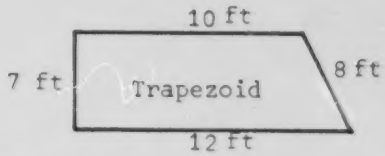
8.



ans. _____

9. 

ans. _____

10. 

ans. _____

We must also be able to find the areas of quadralaterals. The formulas change with the shape of the quadrilaterals.

Since we know a square has four equal sides and all angles are equal, we simply square the length of the side and get the Area. (area is always expressed square units)

Example:

$A = S^2$
$A = 6ft \times 6ft$
$A = 36ft^2$

Problems: Find the area of these squares.

11.

ans._____

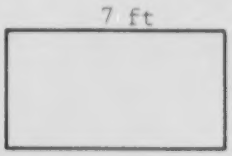
12.

ans._____

Area of a Rectangle

The rectangle has two different lengths of sides. One side is called the length and the other is called the width. If we multiply these two we come up with square units of the figure.

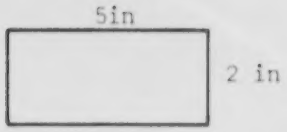
Example:



$A = L \times W$
$A = 7ft \times 5ft$
$A = 35ft^2$

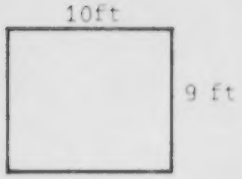
Find the Area of these rectangles:

13.



ans._____

14.

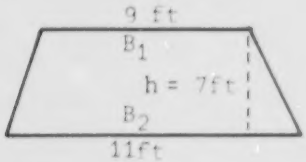


ans._____

Area of a Trapezoid

Since the only constant in a trapezoid is that the two bases are always parallel, we must have a more complicated formula to figure the area of a trapezoid.

Example:



$$A = \left(\frac{B_1 + B_2}{2}\right) \times h$$

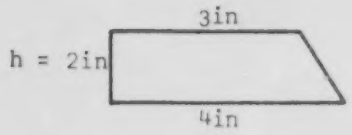
$$A = \left(\frac{9ft + 11ft}{2}\right) \times 7ft$$

$$A = 10ft \times 7ft$$

$$A = 70ft^2$$

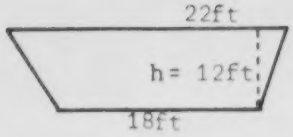
Problems:

15.



ans._____

16.



ans._____

CIRCLES

A circle is a figure with all the points around the outside the same distant from a center point.

Example:

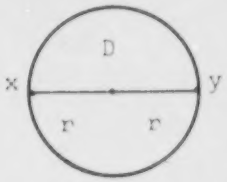
a, b, c, d, e & f

are equal in length.

This distance from the center point to a point on the outside is called the radius.

The straight distance from one side of a circle to the other passing through the center is called the diameter. The diameter is twice the radius.

Example:



$D = 2r$

We have a mathematical constant that we use with circles. This is called pi (the symbols is π). π never changes in value and is the circumference (distance around the outside or perimeter) of a circle divided by the diameter of the circle.

$$\pi = \frac{\text{circumference}}{\text{diameter}}$$

This may be expressed as a fraction or as a decimal

$\frac{22}{7}$ or 3.1416

Circumference of a Circle

Formula: $C = 2\pi r$ or πD

Circumference = 2 π radius or π Diameter

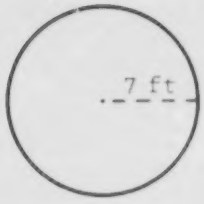
Example:

$C = 2\pi r$
$C = 2 \times 3.14 \times 3ft$
$C = 6.28 \times 3ft$
$C = 18.84ft$

or $C = \pi D$
$C = \frac{22}{7} \times 6ft$
$C = 18.85ft$

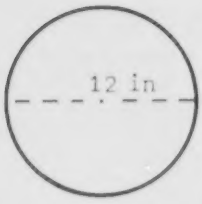
Problems: Find Circumference

17.



ans._____

18.



ans._____

Area of a Circle

The area of a circle is π times the radius squared, or π times the diameter squared divided by 4.

Formula: $A = \pi r^2$

$A = \pi \frac{D^2}{4}$

Example:

$A = \pi \frac{D^2}{4}$

$A = 3.14 \times \frac{8ft \times 8ft}{4}$

$A = 3.14 \times 16ft^2$

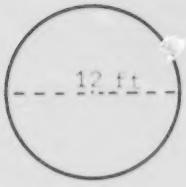
$A = 50.24ft^2$

Problems: Find the Area

19.

ans._____

20.



ans._____

VOLUMES

We must know how much sewage each treatment unit will hold. This is called the volume. Volume is expressed in cubic units i.e. cubic inches (in^3), cubic feet (ft^3), cubic yards (yds^3), and cubic meters (m^3), gallons, etc.*

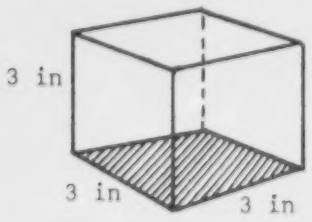
The volume of a solid figure is the number of cubic units of space the figure occupies or in our case the volume of liquid it will hold.

In this section we will concern ourselves with six of the most common solids.

1. Cube
2. Rectangular solid
3. Triangular solid
4. Trapezoidal solid
5. Cylinder
6. Cone

The basic principle of finding the cubic units in a solid is:

Find the square units of area in the figure that carries the name of the solid figure and multiply that by the third dimension.



CUBE

Shaded area is the base.

Since the Base is a square we find the area by multiplying length by width or 3in x 3in. This results in 9 in^2.

To find the volume we multiply the 9 in^2 by 3 in, the height, giving us 27in^3.

*This superscript or exponent ^{3} means multiplying units x units x units and is called cubing.

For example: m^3 = m x m x m = meter x meter x meter

The simplest Formula for volume of a cube is: $V = S^3$ (or $V = S \times S \times S$)

Example: The Volume of a 4 inch cube is

$V = S \times S \times S$

$V = 4in \times 4in \times 4in$

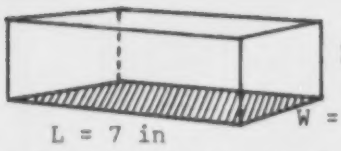
$V = 64\ in^3$

Find the volume of the following:

1. Volume of a 7ft cube = _____

2. Volume of a 2 yd cube = _____

RECTANGULAR SOLID



Shaded area is the base

Volume of a rectangular solid is Area of Base times the height.

The area of the base is Length times Width which we multiply by the Height to get the Volume.

The formula then is $V = L \times W \times H$

From the figure above $V = 7in \times 3in \times 4in$

$V = 84\ in^3$

Find the Volume of the following Rectangular Solids:

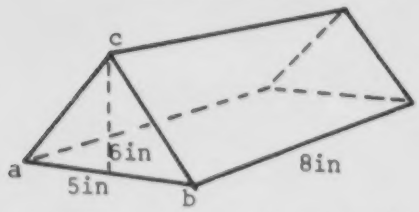
3. L = 5 yds
   W = 2 yds
   H = 3 yds

ans._____

4. L = 6 ft
   W = 4 ft
   H = 3 ft

ans._____

TRIANGULAR SOLID



The base of this solid would be the triangle formed by points a, b & c.

From a previous lesson we know the area of a triangle is ½ the base times the Altitude. In the figure, 5in is the base and 6in is the Altitude.

The Area then is $\frac{5in}{2} \times 6in$

Area is 15 in^2

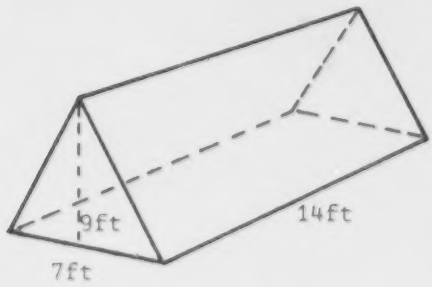
Formula: Volume = ½ base x height x length

$$V = \frac{5in}{2} \times 6in \times 8in$$

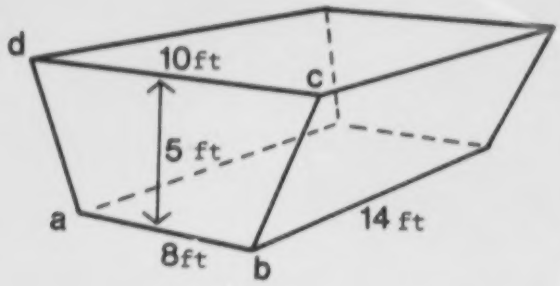
$$V = 15\ in^2 \times 8in$$

$$V = 120\ in^3$$

5. Find Volume of this figure:



TRAPEZOIDAL SOLID



The base of this solid would be a trapezoid formed by points a, b, c & d.

From a previous lesson we know the area of a trapezoid = $\left(\frac{\text{base}_1 + \text{base}_2}{2}\right)$ x height

$$A = \frac{10\text{ft} + 8\text{ft}}{2} \times 5\text{ ft}$$

$$A = 45\text{ ft}^2$$

Formula for Volume = $\left(\frac{\text{base}_1 + \text{base}_2}{2}\right)$ x height x length

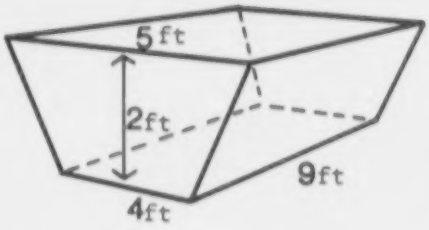
$$V = \left(\frac{10\text{ft} + 8\text{ft}}{2}\right) \times 5\text{ft} \times 14\text{ft}$$

$$V = 9\text{ft} \times 5\text{ft} \times 14\text{ft}$$

$$V = 45\text{ft}^2 \times 14\text{ft}$$

$$V = 840\text{ ft}^3$$

6. Find the Volume of this solid.



ans._____

CYLINDER

The base of this solid would be a circle.

The r representing the radius and h representing the height.

We know that the Formula for area of a circle is πr^2.

$$A = \pi r^2$$

$$A = 3.14 \times (2in)^2$$

$$A = 12.56\ in^2$$

Formula for Volume of a cylinder $= \pi r^2 \times h$

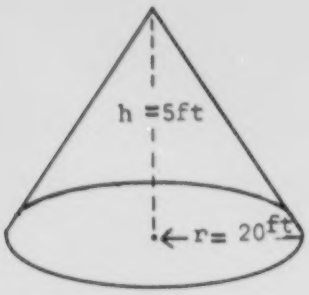
$$V = 3.14 \times (2in)^2 \times 6in$$

$$V = 75.36\ in^3$$

7. Find the volume of a cylinder when the measurments are:

r = 5ft H = 11ft

ans._____



CONE

The base of this solid is a circle. The r represents the radius of the circle and the h the height of the cone.

We know that the Formula for the area of a circle is πr^2

$$A = \pi r^2$$

$$A = 3.14 \times (20ft)^2$$

$$A = 3.14 \times 400ft^2$$

$$A = 1256\ ft^2$$

The Formula for Volume of a cone is $\frac{1}{3} \pi r^2 h$

$$V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{1}{3} \times 3.14 \times (20ft)^2 \times 5ft$$

$$V = \frac{1}{3} \times 1256\ ft^2 \times 5ft$$

$$V = \frac{1}{3} \times 6280\ ft^3$$

$$V = 2093.3\ ft^3$$

8. Find the volume of a cone when the measurements are:

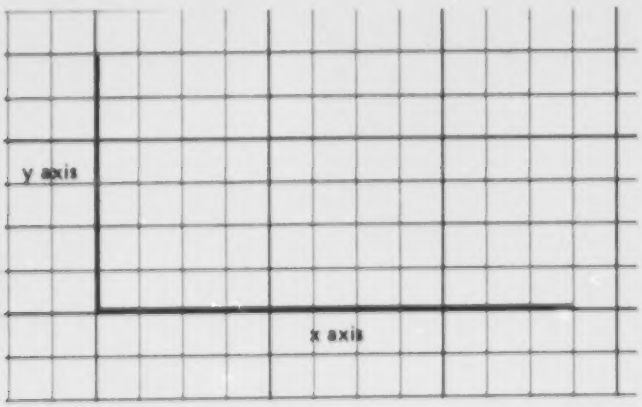
r = 30 ft. h = 54 ft.

GRAPHS

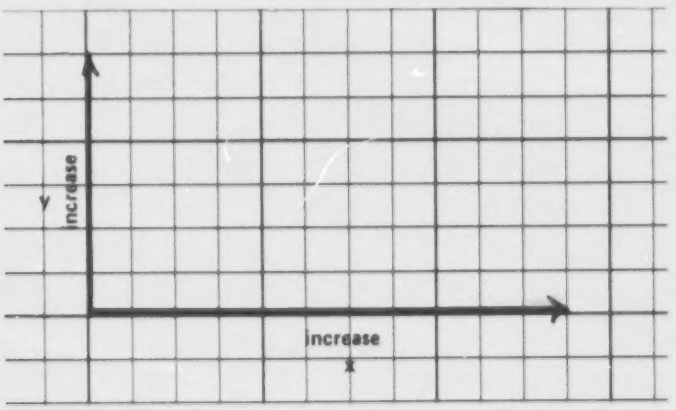
A graph is a diagram or a chart which visually shows a series of changes. These changes may be indicated by curves, lines, dots, dashes, bars, etc. This method of representation indicates how things are changing in proportion to each other.

In this case, we are going to confine ourselves to preparing and understanding line graphs. They are easy to prepare and understand. The flow charts in our treatment plants are line graphs. Charts or graphs which are drawn automatically have the advantage of making and recording continuous measurements. The graphs which we will prepare must be done manually by recording intermittent values, plotting them, and then drawing lines between these points.

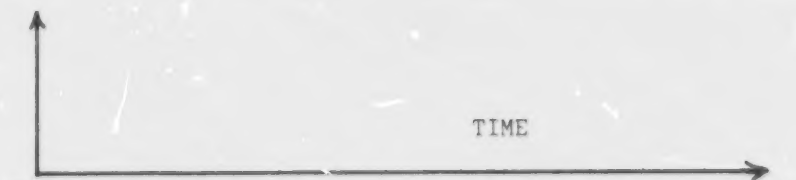
These graphs will meet two conditions. The points to be recorded will be measured in two directions, one measurement horizontally across the paper (the x axis), and the other vertically up and down the paper (y axis). The lines of the graph paper will be the same distance apart both across and up and down the paper.



The size of the numbers will increase as we move up from the point where the x and y axes meet or to the right of where the x and y axes meet.



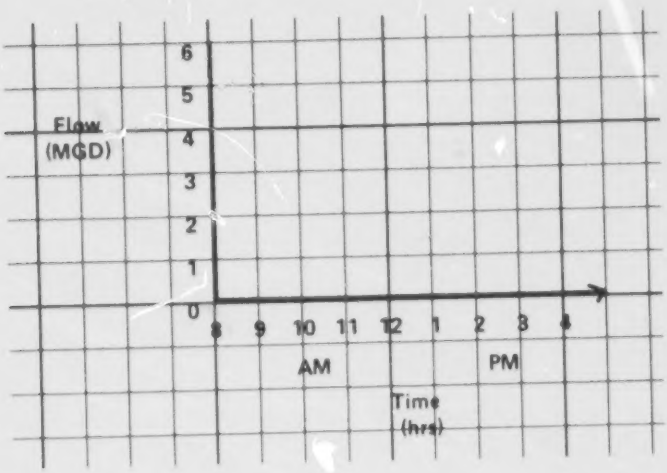
When one of the measurements to be plotted is time, time will be plotted on the horizontal or x axis. Time will start at the point where the y axis meets the x axis and progress toward the right.



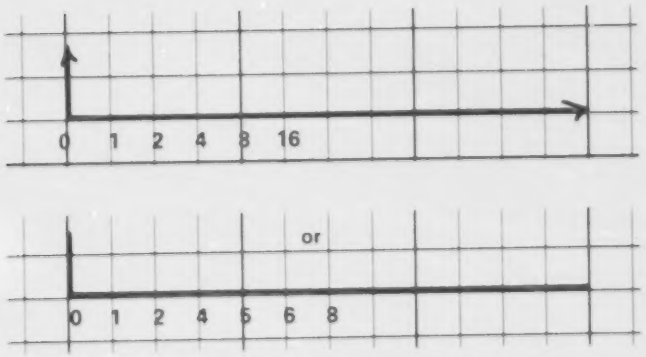
Mon	Tues	Wed	Thur	Fri	Etc.
Jan	Feb	March	April	May	Etc.
0 min	10 min	20 min		30 min	Etc.

The scales of the graph should be meaningful, accurate, and easily understood.

To plot flow against time, for example, we might use values such as these:

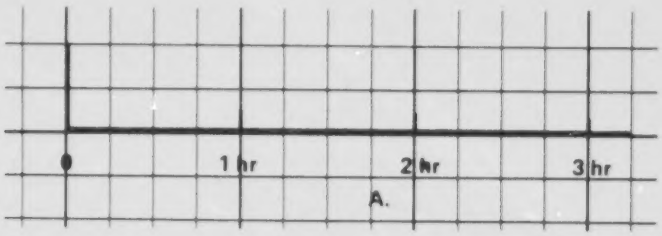


DO NOT vary the units of the scales like this:



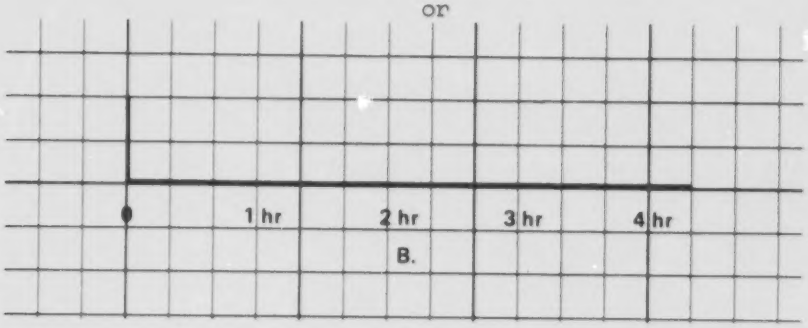
Make your numbers large enough for easy interpretation. Use scales which permit you to easily determine values between the recorded numbers. Use scales in keeping with the numbers and values actually encountered.

For example:



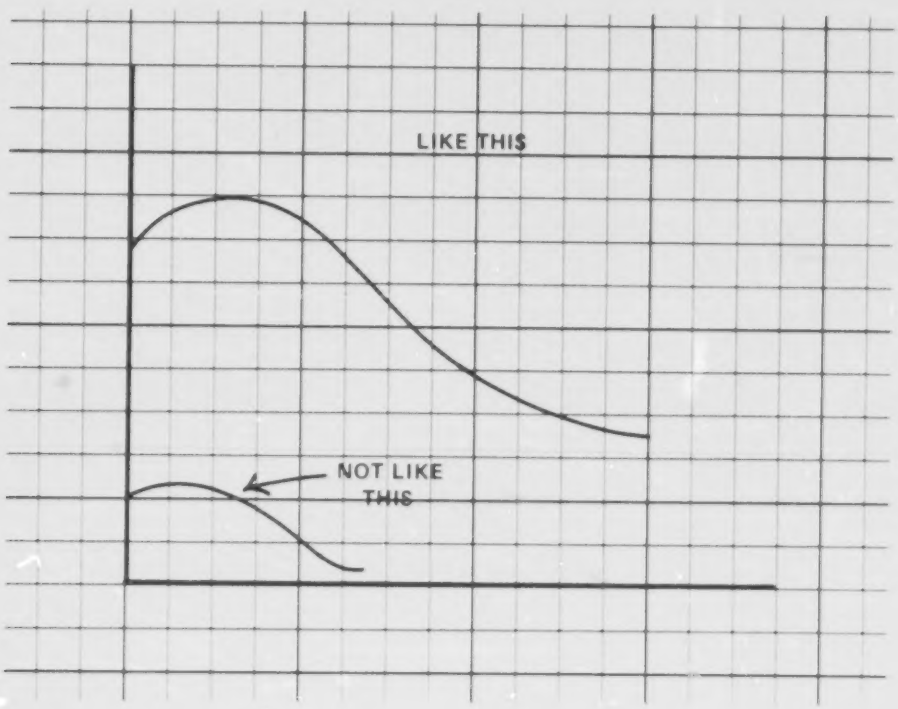
A.

or



B.

Prepare your scales so that the range of recorded values will occupy the major portion of your graph.



Example #1

The flow in your plants is as follows:

Time (hr)	Flow (MGD)
8 A.M.	1.2
9 A.M.	1.4
10 A.M.	1.8
11 A.M.	2.9
12 Noon	3.1
1 P.M.	2.8
2 P.M.	2.0
3 P.M.	1.7
4 P.M.	1.5

Prepare your graph by using the following steps:

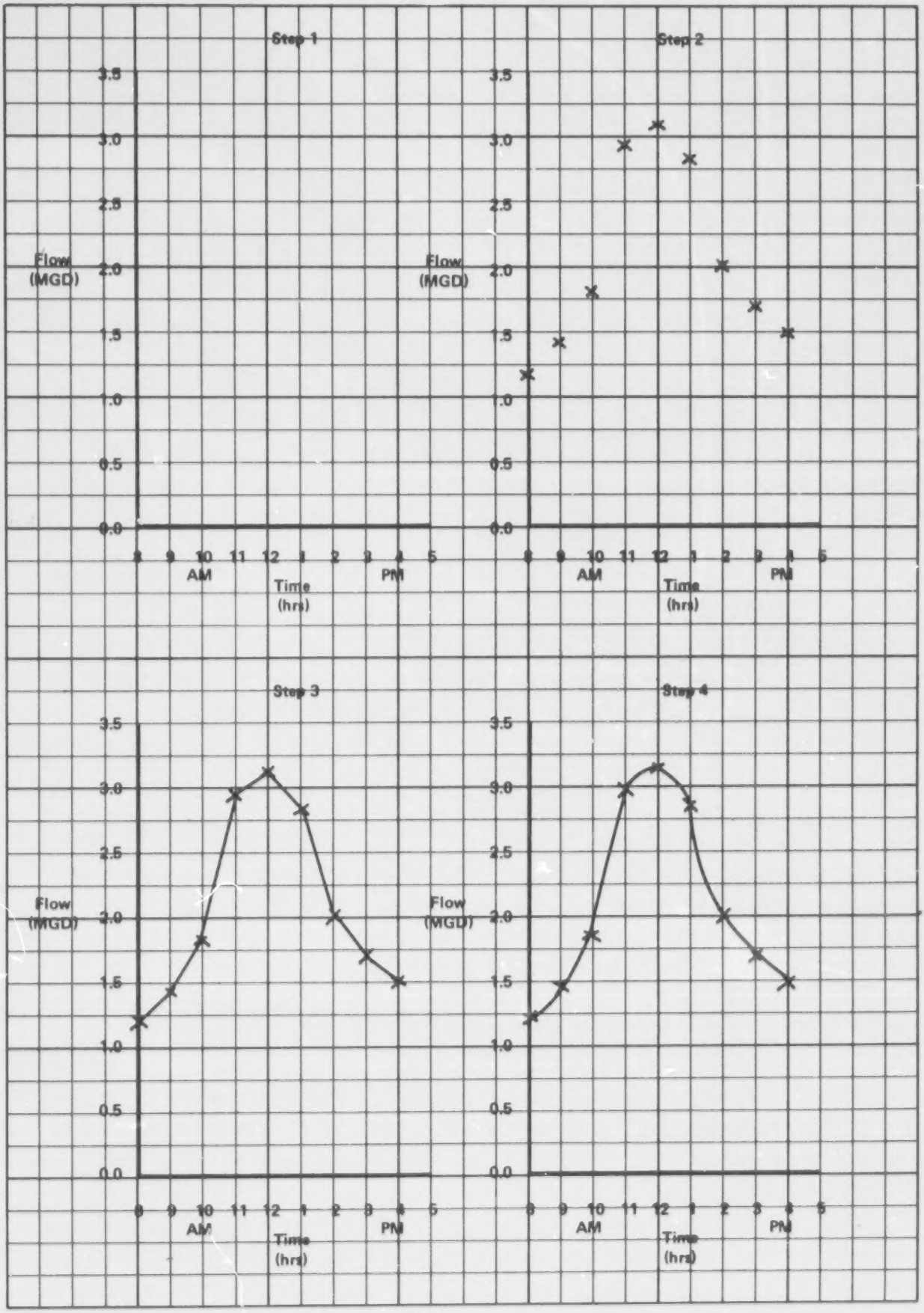
Step #1. Since time is to be plotted on the x axis, determine the scale of values for both time and flow.

Step #2. Plot the points from the above data.

Step #3. Connect the points with straight lines.

Step #4. More meaningful results may be obtained by drawing a smooth curve between the points.

Step #5. Interpret the graph from Step #4.


Step 1
Step 2
Step 3
Step 4
Flow (MGD)
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
8 9 10 11 12 1 2 3 4 5
AM
PM
Time (hrs)

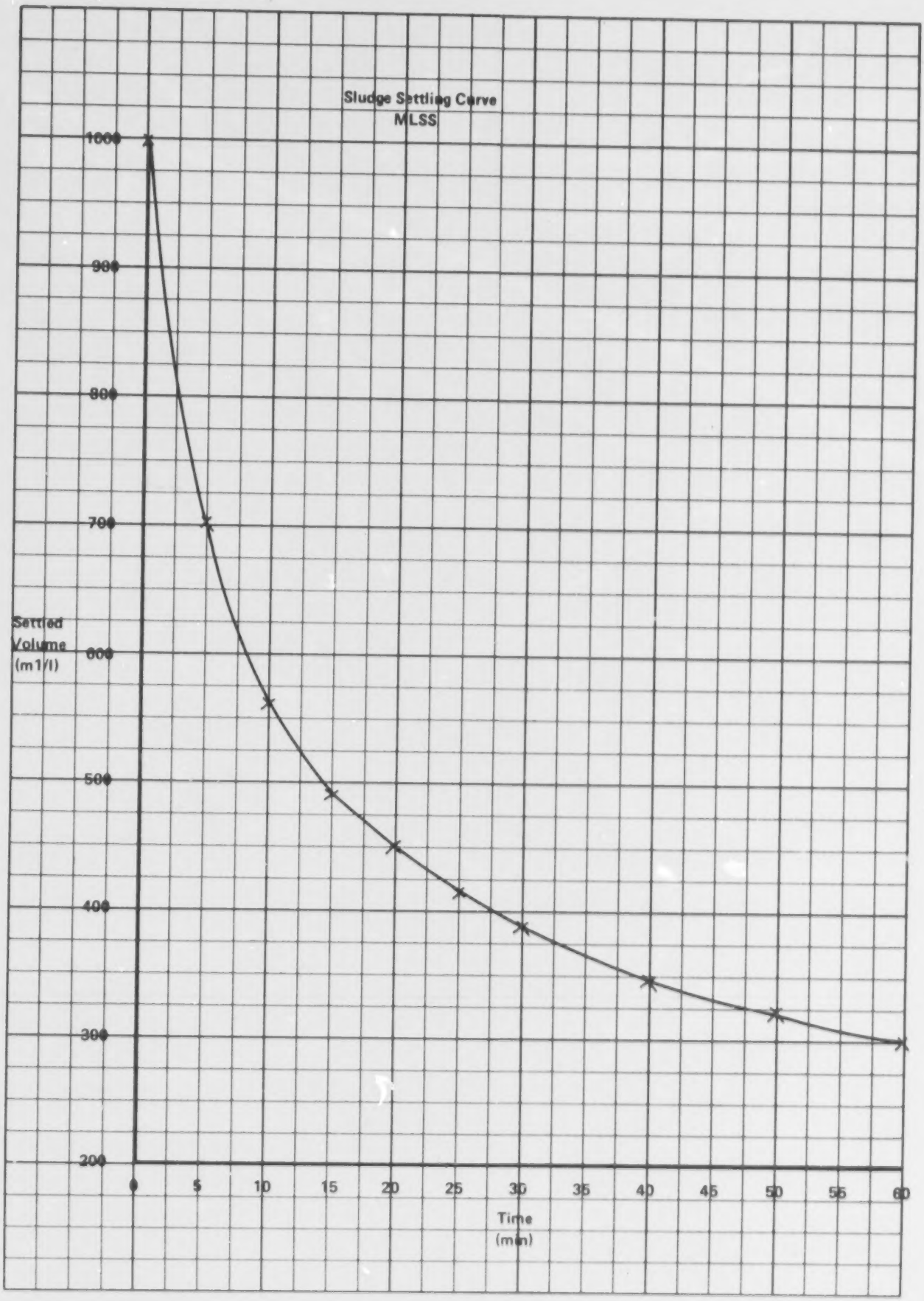
Step #5. Interpretations

(a) When does peak flow occur? Ans. 12 Noon
(b) What is the flow at 9:30? Ans. 1.6 MGD
(c) How long does the peak flow last? Ans. 2 Hrs.
(d) When does the rate of flow increase the fastest? Ans. Between 10 and 11 A.M.

Example #2. A 1 (one) liter sample of mixed liquor suspended solids was taken to the laboratory and allowed to settle in a 1 (one) liter graduated beaker. The volume of settled sludge was measured at 5 (five) minute intervals for the first 30 minutes and every 10 minutes from 30 minutes to 1 (one) hour. The volumes recorded below are plotted on the following page.

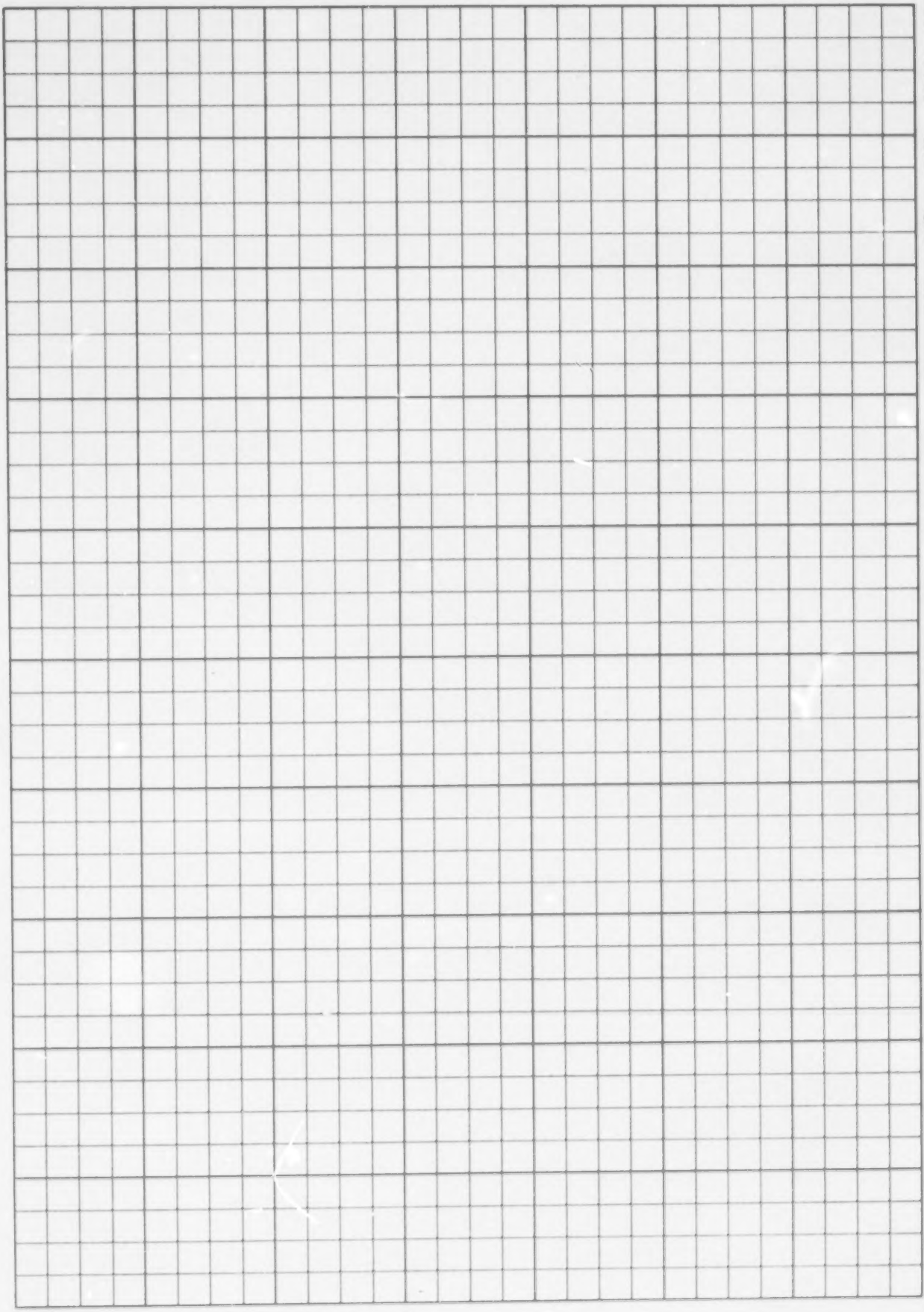
Settling Time (min)	Sludge Volume (ml/l)
0	1000
5	700
10	560
15	490
20	450
25	410
30	390
40	350
50	330
60	305

This is a typical curve of good settling activated sludge.


Sludge Settling Curve
MLSS
1000
900
800
700
600
500
400
300
200
Settled Volume (ml/l)
0
5
10
15
20
25
30
35
40
45
50
55
60
Time (min)

Problem: Graph the following flow data:

TIME	FLOW MGD
8:00 A.M.	3.1
9:00 A.M.	3.2
10:00 A.M.	3.3
11:00 A.M.	3.7
12:00 NOON	3.8
1:00 P.M.	3.7
2:00 P.M.	3.4
3:00 P.M.	3.6
4:00 P.M.	3.7
5:00 P.M.	3.9
6:00 P.M.	4.0
7:00 P.M.	4.1
8:00 P.M.	4.4
9:00 P.M.	4.3
10:00 P.M.	3.8
11:00 P.M.	3.5
12:00 P.M. Midnight	3.2



BASIC MATHEMATICS REVIEW

1) ADDITION

$$\begin{array}{r} 231 \\ 650 \\ 420 \\ 90 \\ +\ 71 \\ \hline \end{array}$$

2) SUBTRACTION

$$\begin{array}{r} 1{,}749 \\ -\ 958 \\ \hline \end{array}$$

3) MULTIPLICATION

$321 \times 71 \times 3 \times 2 =$

4) DIVISION

$1152 \div 64 \div 9 =$

5) SQUARE

12 Feet =

6) CUBE

7 Inches =

CHANGE TO MIXED NUMBERS

7) $\frac{13}{8} =$

8) $\frac{14}{4} =$

9) $\frac{7}{3} =$

REDUCE TO LOWEST TERMS

10) $\frac{15}{45} =$

11) $\frac{216}{324} =$

ADD

12) $\frac{1}{2}$

$\frac{1}{3}$

$+ \frac{3}{5}$

13) $\frac{1}{6}$

$\frac{3}{12}$

$+ \frac{9}{36}$

SUBTRACT

14) $\frac{3}{5}$

$- \frac{7}{12}$

15) $\frac{1}{12}$

$- \frac{1}{25}$

16) MULTIPLY

$\frac{1}{5} \times \frac{3}{7} =$

17) DIVIDE

$\frac{1}{6} \div \frac{7}{10} =$

CHANGE FRACTIONS TO DECIMALS

18) $\frac{1}{12} =$

19) $\frac{7}{8} =$

CHANGE DECIMALS TO FRACTIONS

20) .75 =

21) .95 =

ADD

22) 12.527 + .099 + 5.21 =

23) 4.008 + 3.072 + .23 =

SUBTRACT

24) 11.5173 - 5.929 =

25) .9 - .17 =

MULTIPLY

26) 7.891 x 4.21 =

27) 5.837 x 2.007 =

DIVIDE

28) 56.482 ÷ 2.71 =

29) 3.36 ÷ 40 =

CHANGE % TO FRACTIONS

30) 17% =

31) $3\frac{1}{3}$ % =

CHANGE % TO DECIMALS

32) $2\frac{1}{4}$% =

33) 16% =

CHANGE FRACTIONS TO %

34) $\frac{4}{5}$ =

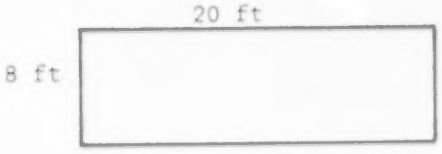
35) $\frac{5}{8}$ =

SOLVE PROPORTION FOR X

36) X : 6 : : 50 : 3 =

37) 3 : X : : 10 : 100 =

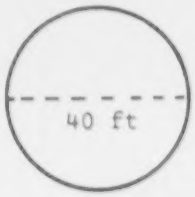
38) A rectangular clarifier has the following surface dimensions:



a. A hand rail has to be installed around the perimeter of this tank. How much railing is required?

b. What is the surface area of this tank?

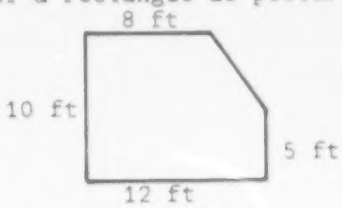
39) A circular clarifier has a diameter of 40 ft. to the outside weir.



a. What is the weir length?

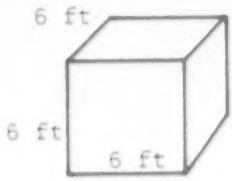
b. What is the surface area of this tank?

40) The cross section of a grit chamber is in the form of a trapezoid on top of a rectangle as pictured.



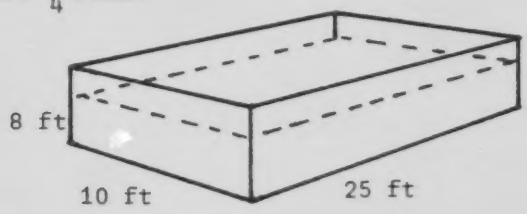
What is the area of this cross section?

41) A grease pit is in the form of a cube:



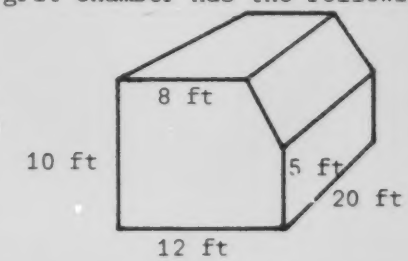
It is required that you record the cu. ft. of grease removed daily. It is filled to the top before it is emptied. What is the volume of grease removed?

42) The primary clarifier is in the form of a rectangle. It is filled $\frac{3}{4}$ full.



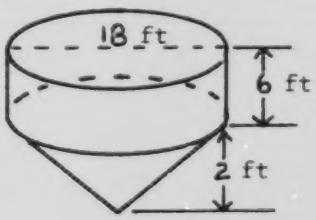
What is the volume of liquid in the tank?

43) Your grit chamber has the following dimensions:



When it is flowing full what is the volume of water in the tank?

44) The primary clarifier is in the form of a cylinder with a cone shaped bottom:



a. The cylindrical section contains raw sewage. What is the volume of the sewage?

b. The conical section contains sludge. What is the volume of the sludge?

FACTOR LABEL METHOD

A useful tool in solving processing problems is the "Factor Label Method." Label factoring is a method of checking the accuracy of formulas before proceeding with the numerical calculations.

Factoring is a term which in the past was commonly used to mean multiplication or division. We are going to apply it to the physical units of measure in our formulas to help us in checking the correctness of the formulas. Since these units invariably take the form of fractions, it is logical that they be considered as fractions and be treated in the same manner.

Terms such as $\frac{\text{lbs}}{\text{day}}$, $\frac{\text{mg}}{1}$, and $\frac{\text{ft}}{\text{sec}}$ can be manipulated in the same way as fractions such as $\frac{1}{2}$, $\frac{3}{4}$, or $\frac{5}{8}$.

A simple example is the conversion of yards to inches.

$$\text{Yds} = \text{yd} \times \frac{3 \text{ ft}}{\text{yd}} \times \frac{12 \text{ in}}{\text{ft}}$$

Let us first solve this equation using only the labels

$$\text{Yds} = \cancel{\text{yd}} \times \frac{\cancel{\text{ft}}}{\cancel{\text{yd}}} \times \frac{\text{in}}{\cancel{\text{ft}}}$$

As in fractions we can cancel the same values which appear both in the numerators and the denominators. In this case yds cancel / yds and ft cancel / ft. Our formula is correct and will change yards to inches and we need only to make the necessary mathematical calculation to give us an answer which is numerically correct.

Thus:

$$\text{Yds} = 3 \times 12 \text{ in} = 36 \text{ in}$$

The process with practice can be done in one operation thus:

$$\text{Yds} = 1 \cancel{\text{yd}} \times \frac{3 \cancel{\text{ft}}}{\cancel{\text{yd}}} \times \frac{12 \text{ in}}{\cancel{\text{ft}}} = 36 \text{ in.}$$

This procedure is called simply the "factor label method" and is the process of treating the labels as fractions and solving for the final units of measure.

Using another example, let us see how this technique can help avoid unnecessary mistakes.

Example:

Convert 2 MGD (million gals per day) to gal/min (gallons per minute)

$$2 \text{ MGD} = \frac{2 \frac{\text{MG}}{\text{day}} \times 1{,}000{,}000 \frac{\text{gal}}{\text{MG}}}{24 \frac{\text{hr}}{\text{day}} \times 60 \frac{\text{min}}{\text{hr}}}$$

Factor the labels: remember in dividing fractions the denominator is inverted (turned upside down) and multiplied times the numerator.

$$\text{MGD} = \frac{\frac{\text{MG}}{\text{day}} \times \frac{\text{gal}}{\text{MG}}}{\frac{\text{hrs}}{\text{day}} \times \frac{\text{min}}{\text{hr}}}$$

$$\text{MGD} = \frac{\cancel{\text{MG}}}{\cancel{\text{day}}} \times \frac{\text{gal}}{\cancel{\text{MG}}} \times \frac{\cancel{\text{day}}}{\cancel{\text{hr}}} \times \frac{\cancel{\text{hr}}}{\text{min}}$$

$$\text{MGD} = \frac{\text{gal}}{\text{min}}$$

$$2 \text{ MGD} = \frac{2 \times 1{,}000{,}000 \frac{\text{gal}}{\text{min}}}{24 \times 60}$$

$$2 \text{ MGD} = \frac{2{,}000{,}000 \frac{\text{gal}}{\text{min}}}{1440}$$

$$2 \text{ MGD} = 1{,}389 \frac{\text{gal}}{\text{min}}$$

Now let us assume that we have carelessly set up our formula the wrong way as follows:

$$2\ \text{MGD} = 2\frac{\text{MG}}{\text{day}} \times 1{,}000{,}000\ \frac{\text{gal}}{\text{MG}} \times \frac{24\ \text{hr}}{\text{day}} \times \frac{60\ \text{min}}{\text{hr}}$$

Factor the label

$$\text{MGD} = \frac{\cancel{\text{MG}}}{\text{day}} \times \frac{\text{gal}}{\cancel{\text{MG}}} \times \frac{\cancel{\text{hr}}}{\text{day}} \times \frac{\text{min}}{\cancel{\text{hr}}}$$

$$\text{MGD} = \frac{\text{gal} \times \text{min}}{\text{day}^2}$$

It is immediately obvious that our answer in gallon minutes per square day is wrong and we must go back and correct our formula. The mathematics are not necessary to check the formula.

Thus the "factor label method" is an excellent check on our formulations.

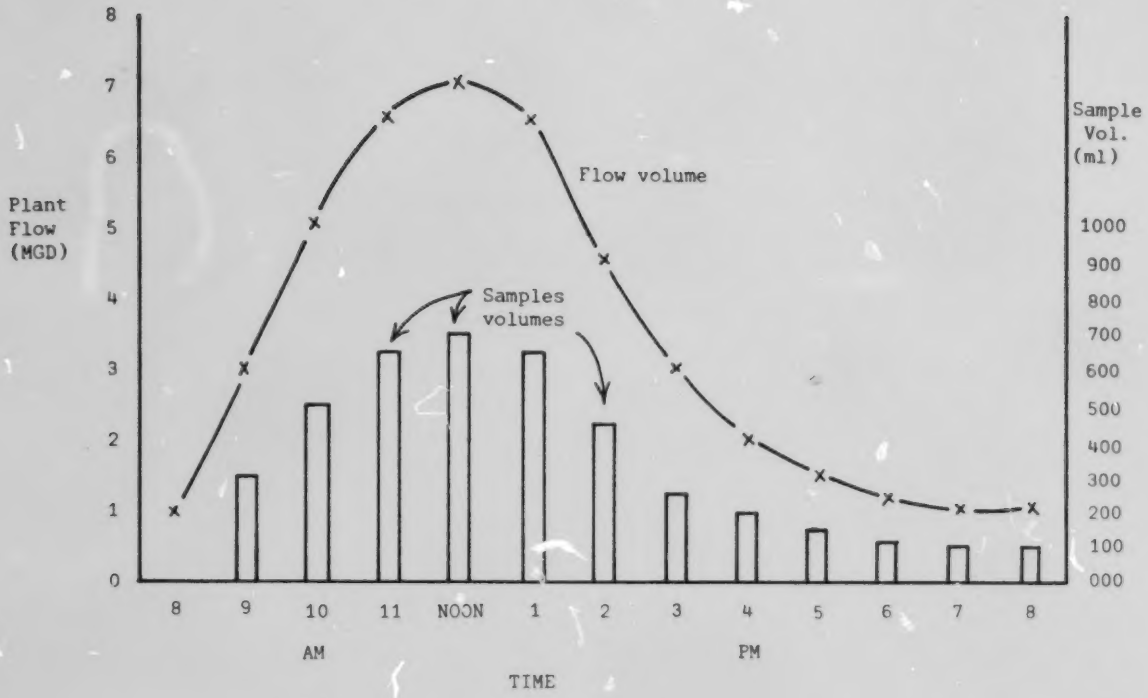
COMPOSITE SAMPLING ACCORDING TO FLOW

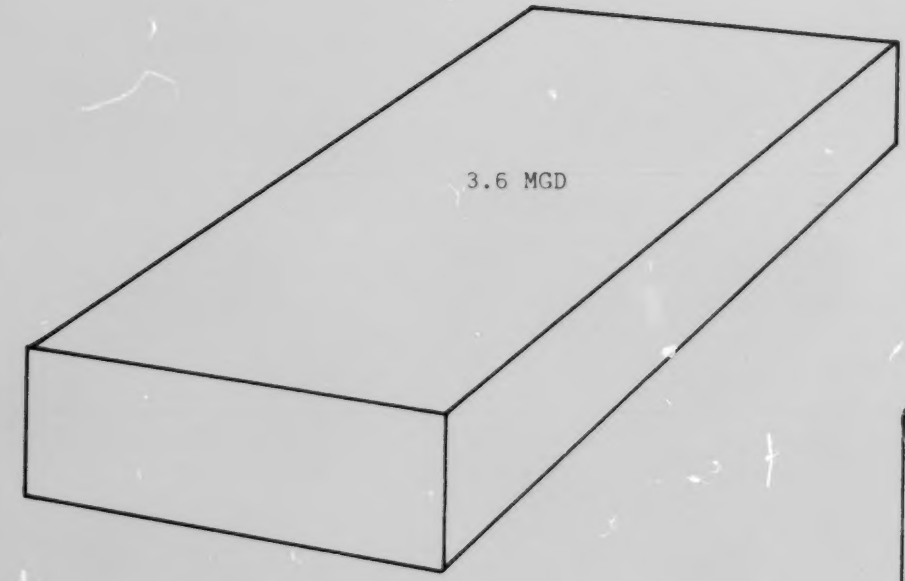
The purpose of a sample is to provide the laboratory with a small volume of sewage which duplicates as nearly as possible the sewage being treated in the plant. The laboratory analyzes the sample for different characteristics and strengths and provides results for the control and evaluation of the plant operations.

The most accurate results would of course be obtained by sampling and analyzing continuously. Since this is neither practical nor economical the next best method is to use a composite sample.

A composite sample is a large sample which is prepared by combining many small samples that are taken at regular intervals throughout the test period. Usually composite samples of eight hours or longer are collected by sampling at hourly intervals. Shorter time periods may be necessary for rapidly changing sewage.

In order for the composite sample to be representative of the sewage being treated, each small sample must be taken in proportion to the flow in the plant. This method is, therefore, called "Composite sampling According to Flow." The following figures in which 100 ml. of sample is collected for each MGD of flow will illustrate the principle of this procedure. It should be understood this is an illustration and the volume of samples will vary with the length of the composite, frequency of sampling, and the volume of flow. The procedure for determining the compositing factor follows the illustrated example.





Average Flow

Composite Sample

DETERMINING THE FACTOR FOR COMPOSITE SAMPLING ACCORDING TO FLOW

Several pieces of information are necessary in order to determine our sampling volume.

1. How large must our composite sample* be in order to provide sufficient material for laboratory analyses? A one gallon bottle is a convenient compositing vessel and will provide enough sample for all analyses. One gallon contains 3785 ml., so if we use a volume of 3600 ml. we will easily fit our sample into the jug with sufficient room for mixing. *Samples should be preserved in an ice chest or refrigerator during the sampling period.

2. What is the average flow in the plant during the sampling period? This is best determined by finding the average flow from plant flow charts of the previous days provided weather conditions have remained approximately the same.

3. What should be the average volume of the small samples for the composite? This is determined by dividing 3600 ml. by the number of samples collected.

4. How frequently should the samples be taken? When conducting composites of eight hours or longer, sampling is usually done hourly unless the sewage exhibits definite changes more frequently. If the influent pH changes markedly every 30-40 minutes for example, then the samples should be collected at 30 minute intervals.

DETERMINING THE FACTOR FOR COMPOSITE SAMPLING ACCORDING TO FLOW

Example #1:

The composite factor must be determined from previous flow data under similar operating conditions.

An eight hour composite starting at 8 A.M. and ending at 4 P.M. is to be made by sampling hourly. The sampling day is clear, and the flow should be slightly lower than the previous day which had a light rain. Flows from the influent chart of the previous day are as follows:

TIME	FLOW (MGD)
8 AM	0.4
9 AM	0.7
10 AM	1.2
11 AM	1.2
NOON	1.2
1 PM	1.1
2 PM	1.0
3 PM	0.8
4 PM	0.7

The sample must approximately fill a one gallon bottle (3600 ml.) (Composite Volume)

Step 1: How many hourly samples are to be taken for this composite period?

Nine (9) - count them!

Step 2: What is the average volume per sample?

$$\text{Average Sample Volume} = \frac{\text{Composite Volume}}{\text{No of samples}}$$

$$\text{Average Sample Volume} = \frac{3600 \text{ ml}}{9}$$

$$\text{Average Sample Volume} = 400 \text{ ml}$$

Step 3: What is the average plant flow from the previous day?

$$\text{Average Plant Flow} = \frac{\text{Sum of hourly flows (MGD)}}{\text{Number of samples}}$$

$$\text{Average Plant Flow} = \frac{0.4+0.7+1.0+1.2+1.2+1.1+1.0+0.8+0.7 \text{ (MGD)}}{9}$$

$$\text{Average Plant Flow} = \frac{8.1}{9}$$

$$\text{Average Plant Flow} = 0.9 \text{ MGD}$$

Step 4: The Compositing Factor is then determined by dividing the average sample volume by the average plant flow.

$$\text{Factor} = \frac{\text{Average sample volume (ml)}}{\text{Average plant flow (MGD)}}$$

$$\text{Factor} = \frac{400 \text{ ml}}{0.9 \text{ MGD}}$$

Factor = 444 or rounded off 440 ml/MGD

Step 5: The Factor is used in making the composite by multiplying the Factor times the flow rate to give each sample size. For example if the flow is 0.6 MGD then the sample size will be:

$$\text{Sample size (ml)} = \text{Flow } (\cancel{\text{MGD}}) \times \text{Factor } \frac{\text{ml}}{\cancel{\text{MGD}}}$$

$$\text{Sample size (ml)} = 0.6 \times 440 \text{ ml}$$

$$\text{Sample size (ml)} = 264 \text{ ml.}$$

The flows and sample volumes of the example composite are illustrated as follows:

TIME	PLANT FLOW (MGD)	x	FACTOR (ml/MGD)	=	SAMPLE VOLUME (ml)
8 AM	0.4	x	440	=	176
9 AM	0.8	x	440	=	352
10 AM	1.1	x	440	=	484
11 AM	1.2	x	440	=	528
NOON	1.3	x	440	=	572
1 PM	1.0	x	440	=	440
2 PM	0.9	x	440	=	396
3 PM	0.8	x	440	=	352
4 PM	0.6	x	440	=	264
			Total Volume of the composite		3564 ml

Sooner or later we are going to encounter rain during a compositing period. The question is: What do we do? Generally speaking if the flow increases more than 25 percent above the normal, the composite should be shortened to the time elapsed or terminated altogether. Greater flows will not lend themselves to good performance evaluations. To handle flow increase up to 25 percent, corrections can be made using the techniques explained in Example #2.

Example #2:

After taking the first three hourly samples you find that the daily flows are running 25 percent higher than previous day from which you determined your sampling factor. This means that your sampling jug

is going to be full three quarters of the way through the composite period.

NO GOOD! WHAT CAN YOU DO?

The total volume of the composite sample will remain the same as calculated if the volume of sample is reduced by the same percentage that the flow increases.

Step 1. In this case, find the total volume of the first three samples and take 25 percent of it.

Volume = 176 ml + 352 ml + 484 ml = 1012 ml.

25% of Volume = 1012 ml x .25 = 253 ml.

Step 2. Shake the composite sample thoroughly and pour out 253 ml. Throw this 253 ml away. This procedure reduces the volume of sample already taken by 25 percent.

Step 3. Reduce the sampling factor by 25 percent and continue sampling using the new factor.

Factor x 0.25 = 440 ml x .25 = 110 ml

Factor - 25% Factor = 440 ml - 110 ml

New Factor = 330 ml

This reduces the remainder of the composite by 25 percent to compensate for the 25 percent increase in flow.

THE FOLLOWING CALCULATIONS ILLUSTRATE THIS PROCEDURE

Time	Calculated Plant Flow (MGD)	Actual Plant Flow (MGD)	x x	Factor Ml/MGD	= =	Sample Volume Ml. Original Factor	Corrected Factor
8 AM	0.4		x	440	=	176	
9 AM	0.8		x	440	=	352	
10 AM	1.1		x	440	=	+484	
						1012 x 75%	= 759
11 AM	(flow increased	1.5	x	330	=		495
NOON	approximately	1.6	x	330	=		528
1 PM	25 percent)	1.3	x	330	=		429
2 PM		1.1	x	330	=		363
3 PM		1.0	x	330	=		330
4 PM		0.7	x	330	=		231
		TOTAL VOLUME OF COMPOSITE					3135 ml.

PERCENT EFFICIENCY

In waste water treatment we are primarily interested in how much material is removed or destroyed by our processes. Our success in reducing pollutants is measured by the difference between what enters our plants and what leaves our plants. Very simply the removal is the measure of our efficiency.

"Removal = Input - Output."

In practice we express this removal as efficiency. We report the efficiency in terms of percentage and so our formula is as follows.

$$\underset{\text{(Removal)}}{\%\ \text{Efficiency}} = \frac{\text{Input - Output}}{\text{Input}} \times 100$$

This is the most basic formula for expressing efficiency in waste water treatment processes and when used consistently, it is entirely adequate in all plant calculations for control, evaluation, and troubleshooting.

The formula as applied to waste water treatment is as follows:

$$\%\ \text{Efficiency} = \frac{\text{Influent - Effluent}}{\text{Influent}} \times 100$$

This formula may be used to determine the efficiency of removal of BOD, COD, suspended solids, suspended volatile solids, phosphate, nitrogen, etc.

As long as the same units of measure are used for both the influent and effluent, any of the various measurements are applicable. For example we can use lbs, mg/l, % volatile solids, etc.

The following examples will clarify these points.

Example #1:

The efficiencies of treatment units, with the exception of anaerobic digesters, are usually measured in terms of BOD or suspended solids removed.

The influent to your plant has a BOD analysis of 150 mg/l and a suspended solids analysis of 180 mg/l. The primary effluent contains 96 mg/l of BOD and 99 mg/l of suspended solids. What is the % Removal or BOD efficiency of this clarifier? What is the % Removal or Suspended Solids efficiency of the clarifier?

Step 1: % Efficiency (BOD) (Removal)

$$= \frac{\text{Influent BOD (mg/l)} - \text{Effluent BOD (mg/l)}}{\text{Influent BOD (mg/l)}} \times 100$$

$$= \frac{150 - 96}{150} \times 100$$

$$= \frac{54}{150} \times 100$$

$$= 36$$

Step 2: % Efficiency (SS) (Removal)

$$= \frac{\text{Influent SS (mg/l)} - \text{Effluent SS (mg/l)}}{\text{Influent SS (mg/l)}} \times 100$$

$$= \frac{180 - 99}{180} \times 100$$

$$= \frac{81}{180} \times 100$$

$$= 45$$

Example #2:

The efficiency of anaerobic digesters is normally calculated by the amount of volatile solids destroyed in the process. If the raw sludge from the primary clarifier (which is fed to the digester) contains 76

percent volatile solids and the digested sludge contains 41.8 percent volatile solids what is the efficiency of the digester?

$$\% \text{ Efficiency} = \frac{\text{Volatile Solids (Raw) } (\%) - \text{Volatile Solids (Digested) } (\%)}{\text{Volatile Solids (Raw)}(\%)} \times 100$$

$$= \frac{76.0 - 41.8}{76.0} \times 100$$

$$= \frac{34.2}{76.0} \times 100$$

$$= 45$$

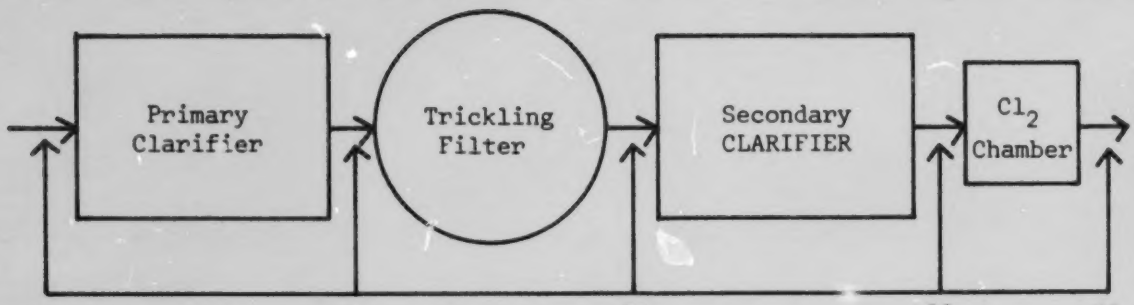
Example # 3:

The efficiency of the overall plant operations as well as each unit should be measured and calculated when the plant is operating well.

When problems develop the trouble may be pinpointed to specific treatment units by reevaluating the efficiency of the various units and comparing these efficiencies to the efficiencies obtained during good operating condition.

During optimum operating conditions, your trickling filter plant was sampled and the following BOD analyses were obtained:

Raw Influent	200 mg/1
Primary Effluent	120 mg/1
T. F. Effluent	50 mg/1
Secondary Effluent	33 mg/1
Final Effluent	30 mg/1



Plant Sampling Points

What is the BOD efficiency or percent BOD removal for the entire plant, the primary clarifier, trickling filter, secondary clarifier and the chlorination chamber.

Step 1:

$$\% \text{ Efficiency (Overall)} = \frac{\text{Influent - Effluent}}{\text{Influent}} \times 100$$

$$= \frac{200 - 30}{200} \times 100$$

$$= \frac{170}{200} \times 100$$

$$= 85$$

Step 2:

$$\% \text{ Efficiency (Primary Clarifier)} = \frac{\text{Influent - Effluent}}{\text{Influent}} \times 100$$

$$= \frac{200 - 120}{200} \times 100$$

$$= \frac{80}{200} \times 100$$

$$= 40$$

Step 3:

$$\% \text{ Efficiency (Trickling Filter)} = \frac{\text{Influent - Effluent}}{\text{Influent}} \times 100$$

$$= \frac{120 - 50}{120} \times 100$$

$$= \frac{70}{120} \times 100$$

$$= 58.3$$

Step 4:

$$\% \text{ Efficiency (Secondary Clarifier)} = \frac{\text{Influent - Effluent}}{\text{Influent}} \times 100$$

$$= \frac{50 - 33}{50} \times 100$$

$$= \frac{17}{50} \times 100$$

$$= 34$$

Step 5:

$$\% \text{ Efficiency (Chlorination Chamber)} = \frac{\text{Influent - Effluent}}{\text{Influent}} \times 100$$

$$= \frac{33 - 30}{33} \times 100$$

$$= \frac{3}{33} \times 100$$

$$= 9.1$$

Problem #1: The results of composite sampling of your plant gave the following suspended solids analyses.

Raw Influent	180 mg/l
Primary Effluent	90 mg/l
Trickling Filter Effluent	45 mg/l
Secondary Effluent	15 mg/l
Final Effluent	10 mg/l

What are the percentage removals in:

(a) Primary Clarifier?

(b) Trickling Filter?

(c) Seconday Clarifier?

(d) Chlorination Tank?

(e) Overall?

LABORATORY CALCULATIONS
SEWAGE SOLIDS

Sewage solids are always reported in terms of milligrams per liter (mg/l). The procedure for converting analytical figures to milligrams per liter (mg/l) is the same for total solids and suspended solids.

The samples are weighed on an analytical balance in the laboratory. These weights are expressed in grams. The first step in these calculations is to convert from grams (gms) to milligrams (mgs). Since there are 1000 mgs. in a gm. the weight (mgs) = 1000 x weight (gms).

Secondly, the samples being analyzed are much smaller than a liter. The weight of the sample must therefore be converted to the weight/liter. This is accomplished by multiplying the weight times $\frac{1000 \text{ (ml)}}{\text{vol. of sample (ml)}}$ (called the dilution factor).

Example 1 - A 50 ml sample of sewage is analyzed in the laboratory for total solids. The evaporating dish alone weighed 38.9652 gms. The dish and the sample after drying weighed 39.0982 gms. What is the total solids analysis in mg/l?

Step 1

Wgt dish + dry sample	=	39.0982 gms
(-) Wgt dish	=	38.9652 gms
Wgt dry sample (gms)	=	0.1330 gms

Step 2

Wgt Sample (mg) = Wgt samples (~~gms~~) x 1000 (mg/~~gm~~)

= 0.1330 x 1000 mg

= 133 mg

Step 3 The weight expressed in steps 1 and 2 are those of the 50 ml sample. This weight must be converted to the weight per liter.

$$\text{Factor} = \frac{1000\ (\cancel{ml})}{\text{Vol. sample}\ (\cancel{ml})}$$

$$= \frac{1000}{50}$$

$$= 20$$

$$\text{Wgt (mg/l)} = \text{Wgt Sample (mg)} \times \text{Factor (sample/l)}$$

$$= 133 \times 20 \text{ mg/l}$$

$$= 2660 \text{ mg/l}$$

Example 2 A 25 ml sample of raw sewage was filtered through a gooch funnel, dried, weighed, ignited and weighed. The analytical weights were as follows:

Gooch + Solids	19.6243 gms
Gooch + Ash	19.6152 gms

What is the analysis of volatile suspended solids?

Step 1

	Gooch + Dry Solids	= 19.6243 gms
(-)	Gooch + Ash	= 19.6152 gms
	Volatile Suspended Solids	.0091 gms

Step 2 Volatile Suspended Solids (mgs) = .0091 $\cancel{gms}$ x 1000 mg/$\cancel{gm}$

= 9.1 mgs

Step 3 $\quad \text{Factor} = \frac{1000 \text{ (ml)}}{\text{Vol. Sample (ml)}}$

$= \frac{1000}{25}$

$= 40$

$\text{V.S.S.} = \text{V.S.S. (mg)} \times \text{Factor (sample/l)}$

$= 9.1 \times 40 \text{ mg/l}$

$= 364 \text{ mg/l}$

The hurrier I go
the behinder I get.

Joe Swift

PROCESSING PROBLEMS
SEWAGE SOLIDS

1. A 40 ml sample of sewage was filtered through a gooch funnel and dried, cooled, and weighed. The weight of the empty gooch was 18.0268 gms and the weight of the gooch and the dry sample was 18.0330 gms.

   (a) What is the weight of the dry sample in gms?

   (b) What is the weight of the sample in milligrams?

   (c) What is the suspended solids content of this sewage in mg/l?

2. If the volatile suspended solids content is 115 mg/l what is the percentage of volatile solids in the sample?

LABORATORY CALCULATIONS
SLUDGE SOLIDS

Sludge solids are measured on a pan balance and the weights are reported in the laboratory to the nearest tenth of a gram (gm). The sludge solids are much more concentrated than sewage solids and for this reason it is not necessary to weigh sludge samples to four decimal places for plant control and evaluation. For control purposes sludge analyses are reported in percentage in either percent solids or percent volatile solids.

Example #1: Fifty (50.) gms of wet sludge was weighed into an evaporating dish. The sample was dried and weighed. The weight of the dish is 41.3 grams and the weight of the dish plus the dry solids was 44.8 gms. What is the percentage of solids in this sample?

Step 1:

	Wgt of dish + dry solids	=	44.8 gms
(-)	Wgt of dish	=	41.3 gms
	Wgt dry solids	=	3.5 gms

Step 2:

$$\text{*Percent Solids} = \frac{\text{Wgt dry solids (gms)}}{\text{Wgt Sample (gms)}} \times 100$$

$$\text{Percent Solids} = \frac{3.5}{50} \times 100$$

$$\text{Percent Solids} = 7.0$$

* It becomes obvious with this formula that if we can use a 100 gm sample then the percent solids is equal to the weight of the dry solids.

$$\text{Percent Solids} = \frac{\text{Wgt dry solids (gms)}}{100 \text{ (gms)}} \times 100$$

(Use 100 gms sample if your laboratory equipment can handle the large size evaporating dishes.)

Example #2 - A 100 gm sample of digested sludge was dried, weighed, ignited, and weighed. The laboratory weights were as follows:

Wgt of Dish + Dry Solids = 41.3 gms

Wgt of Dish + Ash = 38.1 gms

Wgt of Dish = 33.3 gms

What are the percent solids? What are the percent volatile solids?

Step 1. Wgt of dish + dry sample = 41.3 gms

(-) Wgt of dish = 33.3 gms

Wgt of dry sample = 8.0 gms

Step 2. $\% \text{ Solids} = \frac{\text{Wgt dry solids (gms)}}{\text{Wgt Sample (gms)}} \times 100$

$= \frac{8.0}{100} \times 100$

$= 8.0$

Step 3. Wgt of dish + dry sample 41.3 gms

(-) Wgt of dish + ash 38.1 gms

Wgt Volatile Solids 3.2 gms

Step 4. $\% \text{ Volatile} = \frac{\text{Wgt Volatile Solids (gms)}}{\text{Wgt dry solids (gms)}} \times 100$

$= \frac{3.2}{8.0} \times 100$

$= 40$

Calculate the percent solids and percent volatile solids for Samples 1 and 2 on the following page.

LABORATORY CALCULATIONS
SLUDGE SOLIDS

Sample		1	2
Wgt of Wet Sludge	gms	100.0	100.0
Wgt Dish + Dry Sludge	gms	43.2	40.7
Wgt Dish Alone	gms	35.9	38.5
Wgt Dry Sludge	gms	7.3	2.2
Percent Solids	$\frac{\text{Wgt Dry Sludge (gms)}}{\text{Wgt Wet Sludge (gms)}} \times 100$		
Wgt Dish + Dry Sludge	gms	43.2	40.7
Wgt Dish + Ash	gms	37.8	39.9
Wgt Volatile Solids	gms	5.4	0.8
Percent Vol.	$\frac{\text{Wgt Vol. Solids (gms)}}{\text{Wgt Dry Sludge (gms)}} \times 100$		

1a _____ 2a _____

1b _____ 2b _____

LABORATORY CALCULATIONS

SEWAGE BOD

There are a number of procedures which are unique to the BOD analysis. These will be explained in the briefest possible manner here but a more complete answer can be obtained from your local engineer or chemist.

1. The dilution factor for converting the sample analysis to mg/l is as follows:

$$\text{Factor} = \frac{\text{Volume of BOD bottle (ml)}}{\text{Volume of Sample (ml)}} = \frac{300 \text{ (ml)}}{\text{Volume of Sample (ml)}}$$

2. The strength of the sodium thiosulfate solution used to titrate the dissolved oxygen is such that a 200 ml sample must be titrated to provide an answer in units of mg/l. We normally titrate 100 ml and double the answer for a simple reason. Many of our BOD bottles contain somewhat less than 300 ml. If we use a 200 ml sample and overrun the end point it may be difficult to titrate and calculate the BOD from the remaining sample say 95 ml for example. By titrating a 100 ml portion we always have another 100 ml available for a repeat titration and calculation if necessary.

3. The D.O. of the dilution water used in the analysis should be measured at the time the samples are set up and after 5 days in the BOD incubator. These D.O.'s should remain constant. A drop in dilution water D.O. of much greater than 0.3 over the 5 day incubation period is indicative of contamination of the dilution water. The dilution water container should be thoroughly cleaned and the water source and reagents checked.

   The initial D.O. used to calculate the BOD should be the 5 day D.O. of the dilution water.

4. For greatest accuracy in BOD's use only those samples which exhibit a D.O. depletion of 2.0 or greater and which exhibit a final D.O of 1.0 or greater.

Example #1 The BOD test was run on a 20 ml sample of sewage. The initial D.O. of the dilution water was 8.4 and, after 5 days in the B.O.D. incubator, was 8.2. The final D.O. of the sample after 5 day incubation was 3.2. What is the BOD of the sample?

Step #1 Dilution Factor $= \frac{300 \text{ (ml)}}{\text{Vol. sample (ml)}}$

$= \frac{300}{20}$

$= 15$

Step #2 BOD is the amount of dissolved oxygen used up by bacteria during their process of breaking down certain organic wastes. The amount of oxygen used up is the difference between the initial D.O. and the D.O. at the end of the incubation period. The initial D.O. is the 5 day D.O. of the dilution water and the final D.O. of the D.O. of the sample at the end of the incubation period.

Thus:

$$\text{D.O. Depletion} = \underset{\left(\begin{smallmatrix}\text{dilution water}\\ \text{5 day}\end{smallmatrix}\right)}{\text{Initial D.O.}} - \underset{\left(\begin{smallmatrix}\text{sample}\\ \text{5 day}\end{smallmatrix}\right)}{\text{Final D.O.}}$$

D.O. Depletion = 8.2 (mg/l) - 3.2 (mg/l) = 5.0 mg/l

Step #3 The BOD of the sample is determined by multiplying the dissolved oxygen depletion times the dilution factor

BOD (mg/l) = D.O. Depletion x Dilution Factor

BOD (mg/l) = 5.0 x 15

BOD (mg/l) = 75

Example #2 The following laboratory analysis sheet is explained as follows:

Laboratory Analysis

	D.O. Dil H_2O (init)	D.O. Dil H_2O (5 day)	10 ml	5 ml	3 ml
Vol. of Sample					
Factor $\frac{300\ (ml)}{\text{Vol. Sample}\ (ml)}$			30	60	100
Final Burette Reading	42.0	38.0	33.4	32.7	30.5
Initial Burette Reading	37.7	33.8	32.7	30.5	27.4
100 ml Titre	4.3	4.2	0.7	2.2	3.1
200 ml Titre (100 Titre x 2) (200 ml Titre=D.O. mg/l)	8.6	8.4	1.4	4.4	6.2
Initial D.O. (mg/l)			8.4	8.4	8.4
Final D.O. (mg/l)			1.4	4.4	6.2
D.O. Depletion (mg/l)			7.0	4.0	2.2
DOD - D.O. Depletion x Factor (mg/l)			210	240	220
Ave BOD (mg/l)				223	

Step #1. Dilution Factor of the three sample dilutions are:

$$\text{10 ml Sample Factor} = \frac{300}{10} = 30$$

$$\text{5 ml Sample Factor} = \frac{300}{5} = 60$$

$$\text{3 ml Sample Factor} = \frac{300}{3} = 100$$

Step #2. Final Burette Reading
(-)Initial Burette Reading
100 ml Titre

42.0	38.0	33.4	32.7	30.5
(-)37.7	(-)33.8	(-)52.7	(-)30.5	(-)27.4
4.3	4.2	0.7	2.2	3.1

Step #3. 100 ml Titre x 2 = 200 ml Titre

The 200 ml titre is the D.O. in mg/l of the sample in each respective BOD bottle.

In the case of the dilution water it is the D.O. of the dilution water before and after 5 days incubation. In the case of the sample bottles it is the amount of D.O. left after 5 days of bacterial digestion.

4.3	4.2	0.7	2.2	3.1
x 2	x 2	x 2	x 2	x 2
8.6	8.4	1.4	4.4	6.2

Step #4. Since the D.O. of the dilution water after 5 days has not dropped more than 0.2 mg/l the dilution water appears satisfactory.

8.6 - 8.4 = 0.2

The 5 day D.O. of the dilution water is then used as the initial D.O. of the samples and is entered as initial D.O.

Step #5. The dissolved oxygen depletion at each of the three dilutions is then calculated by subtracting their final D.O.'s from this initial D.O. of 8.4.

10 ml	5 ml	3 ml
8.4	8.4	8.4
-1.4	-4.4	-6.2
7.0	4.0	2.2

Step #6. At each of these dilutions there is a depletion in dissolved oxygen of 2.0 or more and also a residual oxygen of 1.0 or more. Therefore each sample dilution should give an accurate BOD.

Step #7. Calculate the BOD for each sample dilution by multiplying each D.O. depletion times their respective dilution factor.

Depletion	(mg/l)	7.0	4.0	2.2
Factor		x 30	x 60	x 100
BOD	(mg/l)	210	240	220

Step #8. The final D.O. of the sample is determined by averaging the accurateBOD's at the various dilutions

$$\text{Ave BOD (mg/l)} \quad \frac{210 + 240 + 220}{3}$$

$$\text{Ave BOD (mg/l)} = \frac{670}{3}$$

$$\text{Ave BOD (mg/l)} = 223$$

Parkinson's Law

Work will always expand
to fill the time available
for its completion.

LABORATORY CALCULATIONS

BOD SEED CORRECTION

The accuracy of the BOD test depends on having a sufficient number of healthy active bacteria present to utilize the dissolved oxygen to break down or destroy the organic waste in the sewage. There are generally two conditions which occur that can reduce the efficiency or number of active bacteria and thereby reduce the accuracy of this test. These conditions are an undesirable pH, ie. less than 6.5 or greater than 7.5 or the presence of chlorine.

The BOD seeding procedure is used if the pH of the sample is less than 6.5 or greater than 7.5, or if the sample has a chlorine residual.

The seeding procedure requires first that the sample must be neutralized and any chlorine residual must be destroyed. The sample is then innoculated with healthy active bacteria. These bacteria are added in the form of a small amount of additional sewage from a source free from toxic waste, which has been incubated for a period of 24 hours, and which has also been filtered to give a homogeneous solution which will not vary from sample to sample. The samples are innoculated by adding this special sewage (seed) to the dilution water. Thus each sample will receive the same type and volume of active bacterial seed.

If we are going to add additional sewage in the form of seed then the depletion of oxygen will be increased by this additional sewage. Our BOD will then be falsely high and we must, therefore, correct for the depletion in oxygen caused by the seed. The seed correction is made in the initial D.O. All additional calculations are subsequently adjusted for the seed by using this corrected initial D.O.

Sufficient active bacteria will be present if we add 0.1 percent of normal active sewage. This amounts to 0.3 ml of seed sewage/BOD bottle. (ie. 300 x .001 = 0.3 ml). It is obvious to those who have conducted BOD tests that 0.3 ml of sewage (seed) will not give an accurate measure of depletion. Ten times this volume of sewage (3.0 ml) is used to obtain an accurate depletion. One tenth of this depletion is then used as the D.O. correction. The procedure is as follows:

Step #1. Add 1 ml of seed sewage to each liter of normal dilution water.

Step #2. Titrate the initial D.O. of this dilution water.

Step #3. Prepare a seed sample as follows:

(a) Place 2.7 ml of seed into the BOD bottle.

(b) Fill the bottle with the seeded dilution water

(c) The total amount of seed in this sample is 2.7 ml added directly, plus 0.3 ml added in the dilution water for a total of 3.0 ml. The volume of seed at 3.0 ml is ten times the amount (0.3 ml) added to the test samples in the dilution water.

(c) The depletion of oxygen in the seed sample is 10 times that which is caused by the seed in the test samples.

Step #4. Incubate the seed sample for 5 days along with the samples being tested. Determine the final D.O. of the seed sample after 5 days.

Step #5. The formula for seed corrected initial D.O. is:

$$\text{Corrected Initial D.O.} = \underset{\text{dilution water}}{\text{Init. D.O.}} - 1/10 \left(\text{D.O. depletion in the seed sample}\right)$$

$$\text{Corrected Initial D.O.} = \text{Init. D.O.} - 1/10 \left(\begin{matrix}(\text{Init. D.O.} - \text{Final D.O.}) \\ (\text{dilution water}) \quad (\text{Seed sample})\end{matrix}\right)$$

For example, if the initial D.O. of the dilution water at the time of preparation of the dilution water was 8.5 and the 5 day D.O. of the seed sample was 4.5, what is the Corrected Initial D.O. to compensate for the seed?

$$\text{Corr. Init. D.O.} = \text{Init. D.O. (dil. water)} - 1/10 \left(\text{Init. D.O. dil water} - \text{Final D.O. seed sample}\right)$$

$$= 8.5 - 1/10\ (8.5 - 4.5)$$

$$= 8.5 - 1/10\ (4.0)$$

$$= 8.5 - 0.4$$

$$= 8.1$$

Step #6. The corrected initial D.O. of 8.1 is used to calculate the individual and average BOD's of all samples using this dilution water.

Using the following laboratory analysis sheet, determine the average BOD of the sample.

	Laboratory Analysis Init D.O. dilution water	Seed Sample	BOD 15	10	5
Vol Sample (ml)					
Factor $\frac{300}{\text{Vol Sample}}$			20	30	60
Final Burette Reading	44.3	30.2	28.0	27.0	24.9
Initial Burette Reading	40.1	28.2	27.0	24.9	22.0
100 ml titre	4.2	2.2	1.0	2.1	2.9
200 ml titre = 100 ml titre x 2	8.4	4.4	2.0	4.2	5.8

Corr. Initial D.O. (mg/l)	8.0	8.0	8.0
Final D.O. (mg/l)	2.0	4.2	5.8
D.O. Depletion (mg/l)	6.0	3.8	2.2
BOD = Depletion x Factor (mg/l)	120	114	132
Average BOD (mg/l)		122	

Step #1. Factors for the three dilutions are:

15 ml sample Factor = $\frac{300}{15} = 20$

10 ml sample Factor = $\frac{300}{10} = 30$

5 ml sample Factor = $\frac{300}{5} = 60$

Step #2. Final Burette Reading
(-)Initial Burette Reading
100 ml titre

44.3	30.2	28.0	27.0	24.9
-40.1	-28.0	-27.0	-24.9	-22.0
4.2	2.2	1.0	2.1	2.9

Step #3. 200 ml titre = 100 ml titre x 2

4.2	2.2	1.0	2.1	2.9
x 2	x 2	x 2	x 2	x 2
8.4	4.4	2.0	4.2	5.8

Step #4. Corrected Init. D.O. = Init D.O. dilution water - 1/10 (Init D.O. dilution water - Final D.O. BOD Seed Sample)

= 8.4 - 1/10 (8.4 - 4.4)

= 8.4 x 1/10 (4.0)

= 8.4 - 0.4

= 8.0

Step #5. Use Corrected Init. D.O. and determine D.O. depletion for each of the three dilutions

Corrected Init D.O. mg/l	8.0	8.0	8.0
(-) Final D.O. mg/l	-2.0	-4.2	-5.8
D.O. Depletion mg/l	6.0	3.8	2.2

Step #6. Calculate BOD for each dilution

BOD (mg/l) = Depletion (mg/l) x Factor

6.0 mg/l	3.8	2.2
x 20	x 30	x 60
120 mg/l	114 mg/l	132 mg/l

Step #7. Determine the final BOD by averaging the individual BOD's.

$$\text{Ave BOD (mg/l)} = \frac{120\ (\text{mg/l}) + 114\ (\text{mg/l}) + 132\ (\text{mg/l})}{3}$$

$$\text{Ave BOD (mg/l)} = \frac{366}{3}$$

$$\text{Ave BOD (mg/l)} = 122$$

B. O. D.
PROBLEMS

Problem # 1 A sample of sewage was set up for B.O.D. analysis. Three dilutions were used for the sample 15 ml, 10 ml, and 5 ml.

(a) What is the dilution factor for each bottle?

(b) The D.O. of the dilution water is 8.2.

If the final D.O. at each dilution are

Dilution	15 ml	10 ml	5 ml
Final D.O.	2.0	3.8	6.2

What are the oxygen depletions at each dilution?

(c) Using their respective oxygen depletions and dilution factors calculate the B.O.D.'s for each dilution.

(d) What is the average B. O. D. of the sample?

Problem # 2

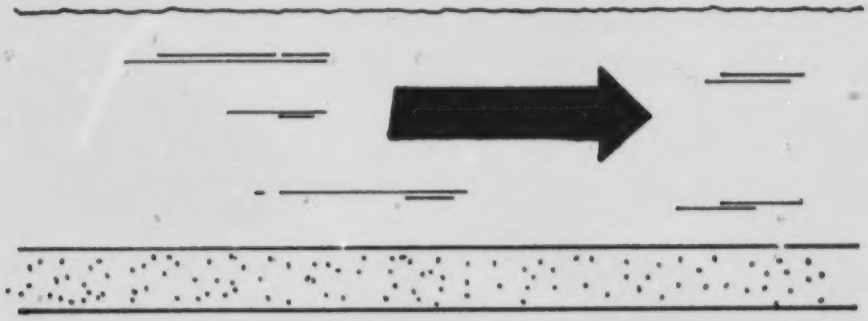
The sample you are analyzing for B. O. D. contained chlorine. The chlorine was destroyed and this dilution water was seeded. The initial D. O. of the dilution water was 8.1. The B.O.D. seed sample after five days had a final D. O. of 3.0. What is the corrected initial D. O. to be used to calculate the sample B.O.D.?

GRIT CHAMBERS

The speed at which sewage flows through sewers, either by gravity or by pumping, is maintained at 2.0 ft/sec or greater. This velocity must be maintained to prevent solids from settling out in the sewer lines resulting in blockages and septic conditions in the collection system.

The grit chamber on the other hand is supposed to reduce this velocity to the point where the heavier abrasive grit settles out and the organic or digestible solids remain suspended and pass through to the succeeding treatment processes. If the size (volume) of the grit chamber is larger than the collecting sewer, then the rate of flow through this unit will be lowered. Both the channel and tank type grit chambers are designed in this manner.

In channel type grit chambers, the sewage flows straight through and therefore, the linear velocity can be measured easily. The optimum velocity in these units is 1 ft/sec.



STRAIGHT CHANNEL GRIT CHAMBER

Flow velocity by measurement:

Upon measuring the length of your grit chamber, you have found it to be 24 ft. Using floating solids or dye, you also found that the sewage passed through the grit chamber in 20 sec.

What is the flow velocity in your grit chamber?

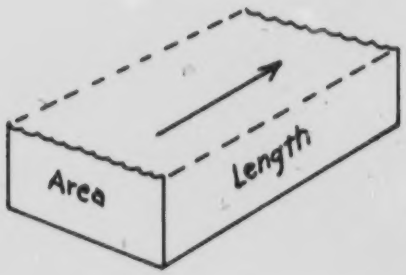
$$\text{Velocity}\left(\frac{\text{ft}}{\text{sec}}\right) = \frac{\text{Length (ft)}}{\text{Time (sec)}}$$

$$\text{Velocity} = \frac{24 \text{ ft}}{20 \text{ sec}}$$

$$\text{Velocity} = 1.2 \frac{\text{ft}}{\text{sec}}$$

Flow velocity by calculation:

The volume of flow through a straight channel grit chamber can be calculated by multiplying the cross-sectional area times the length of the chamber.



Volume = Area x length

From this we can reason that if we know the rate of flow (that is the volume of flow per second) and divide it by the cross-sectional area of the sewage flowing in the channel, we can find the flow velocity.

$$\text{Flow Velocity} = \frac{\text{Rate of Flow } \left(\frac{ft^3}{sec}\right)}{\text{Cross-sectional area of sewage } (ft^2)}$$

$$\text{Flow Velocity} = \frac{ft}{sec}$$

The following is the procedure used to convert our usual flow measurement of MGD to $\frac{ft^3}{sec}$.

$$\text{Rate of flow } \frac{ft^3}{sec} = \frac{1 \frac{MG}{D} \times 1{,}000{,}000 \frac{gal}{MG}}{7.48 \frac{gal}{ft^3} \times 24 \frac{hr}{day} \times 60 \frac{min}{hr} \times 60 \frac{sec}{min}}$$

*Factor the label:

$$\text{Rate of Flow} = \frac{MG}{D} \times \frac{gal}{MG} \times \frac{ft^3}{gal} \times \frac{day}{hr} \times \frac{hr}{min} \times \frac{min}{sec} = \frac{ft^3}{sec}$$

$$\text{Rate of flow } \left(\frac{ft^3}{sec}\right) = \frac{1{,}000{,}000}{7.48 \times 24 \times 60 \times 60} \frac{ft^3}{sec}$$

$$\text{Rate of flow} = 1 \text{ MGD} = 1.55 \frac{ft^3}{sec}$$

If the flow entering the plant is 4 MGD, what is the rate of flow in $\frac{ft^3}{sec}$?

Rate of flow = $4.0 \cancel{MGD} \times 1.55 \frac{\frac{ft^3}{sec}}{\cancel{MGD}}$

Rate of flow = $6.20 \frac{ft^3}{sec}$

EXAMPLE: The flow to your plant is 6 MGD. Your grit chamber is 4 ft. wide and the sewage is flowing at a depth of 9 in. Find the flow velocity.

$$\text{Flow Velocity} = \frac{\text{Rate of Flow } \left(\frac{ft^3}{sec}\right)}{\text{Cross sectional area of sewage } (ft^2)}$$

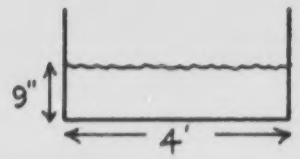
Step 1: Rate of flow = $6.0 \cancel{MGD} \times 1.55 \frac{\frac{ft^3}{sec}}{\cancel{MGD}}$

Rate of flow = $9.3 \frac{ft^3}{sec}$

Step 2: Area of sewage = L (ft) x W (ft)

Area = 4 ft x $\frac{3}{4}$ ft

Area = $3\ ft^2$



Step 3: $$\text{Flow Velocity} = \frac{\text{Rate of Flow } \left(\frac{ft^3}{sec}\right)}{\text{Area of sewage } (ft^2)}$$

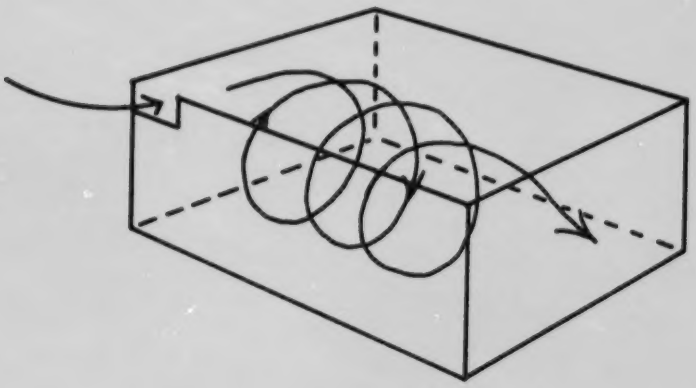
$$\text{Flow Velocity} = \frac{9.3 \left(\frac{ft^3}{sec}\right)}{3\ ft^2}$$

$$\text{Flow Velocity} = 3.1\ \frac{ft}{sec}$$

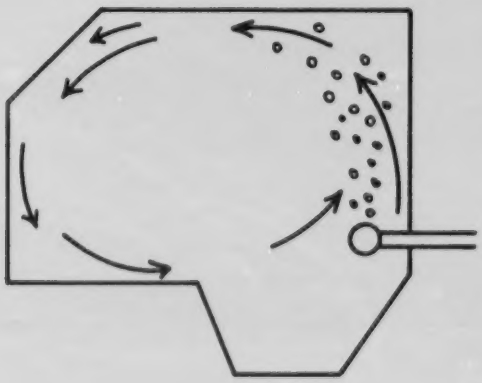
Linberg's "Principle of the Vital Few"

90% of the work is accomplished
by 10% of the workforce.

In tank type grit chambers, the sewage flows through the unit in a circular motion much like the coils of a spring.



This circular motion is caused by the way the sewage enters the grit chamber, the shape of the tank, and in the case of aerated grit chambers, by the flow of air.



For this reason, it is difficult to find the linear velocity, and detention time is used as the measure of best separation.

The optimum detention time in these units is in the range of 30 sec. to 90 sec.

TANK TYPE GRIT CHAMBER

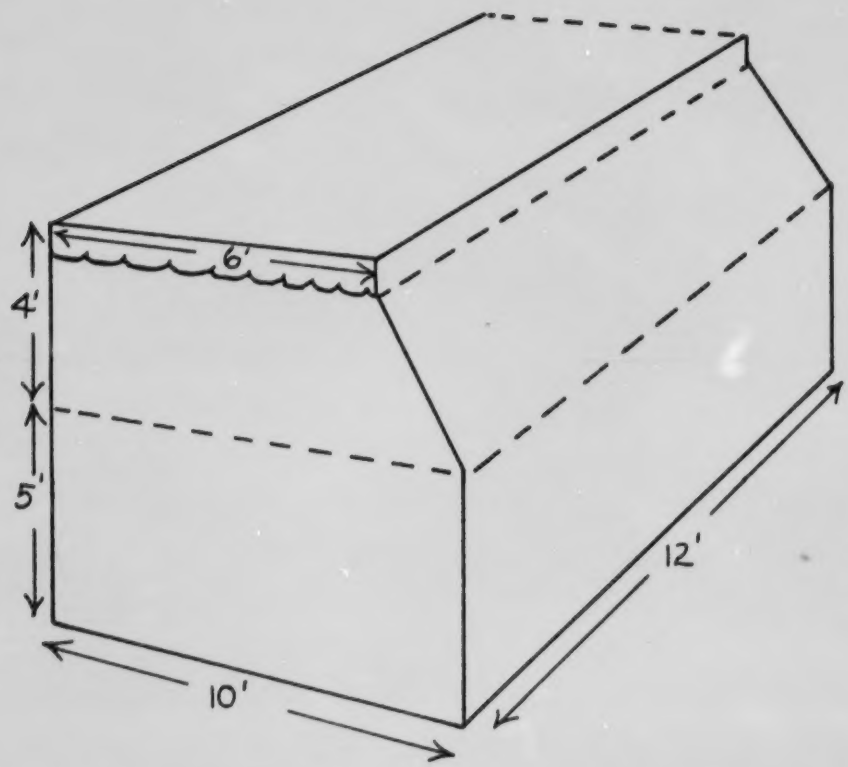
The detention time is the time it takes to fill the tank.

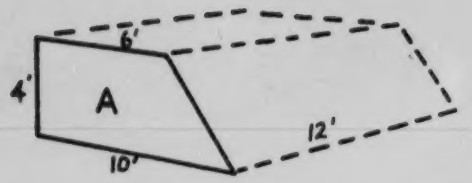
To put it another way, it is the time it takes for a single particle to flow through the treatment unit.

$$\text{Detention Time (sec.)} = \frac{\text{Volume of Grit Chamber } (ft^3)}{\text{Rate of Flow } \left(\frac{ft^3}{sec}\right)}$$

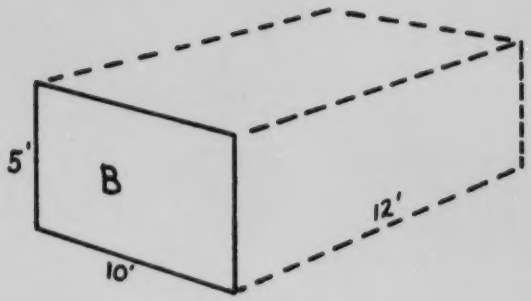
(Factor the label: D.T. (sec) $= \frac{ft^3}{\frac{ft^3}{sec}} = \cancel{ft^3} \times \frac{sec}{\cancel{ft^3}} =$ sec)

Step 1: Volume of Grit Chamber





Trapezoidal Section A



Rectangular Section B

Vol. of Grit Chamber = Area Trapezoid A x L + Area Rectangle B x L

Vol. of Grit Chamber = $\left(\frac{10\text{ft} + 6\text{ft}}{2}\right)$ x 4 ft x 12 ft + (5ft x 10ft) x 12 ft

Vol. of Grit Chamber = (32 ft^2) x 12 ft. + (50 ft^2) x 12 ft

= 384 ft^3 + 600 ft^3

= 984 ft^3

Step 2: Rate of Flow

If the flow to the plant is 12 MGD, what is the rate of flow in $\frac{ft^3}{sec}$?

1 MGD = 1.55 $\frac{ft^3}{sec}$

$$\left(\text{Factor the label: Rate of Flow} = \frac{\text{MGD} \times \frac{ft^3}{sec}}{\text{MGD}} = \frac{ft^3}{sec}\right)$$

$$\text{Rate of Flow} = 12 \times 1.55 \frac{ft^3}{sec}$$

$$12 \text{ MGD} = 18.6 \frac{ft^3}{sec}$$

Step 3:

$$\text{D.T.} = \frac{\text{Vol. of Grit Chamber } (ft^3)}{\text{Rate of Flow } \left(\frac{ft^3}{sec}\right)}$$

$$= \frac{984 \text{ } ft^3}{18.6 \frac{ft^3}{sec}}$$

$$= 53.5 \text{ sec}$$

Problem #1

Your grit chamber is 30 feet long. If dye will pass through this unit in 24 seconds, what is the flow velocity through the chamber?

Problem #2

Your grit chamber is 3 feet wide and the depth of sewage flowing in the channel is 18 inches. The flow through your plant averages 2.32 MGD.

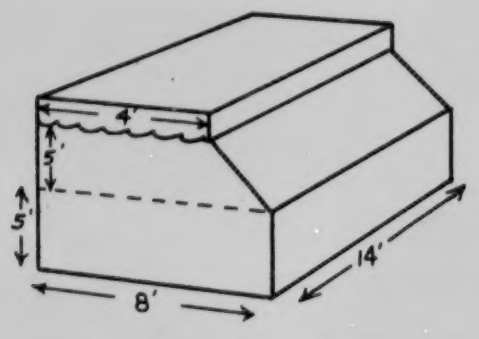
(a) What is the Rate of Flow in $\frac{ft^3}{sec}$?

(b) What is the cross sectional area of sewage flowing in the channel?

(c) What is the velocity of sewage flow through the grit chamber?

Problem #3

Your grit chamber has the following dimensions:



(a) What is the volume of the rectangular section in ft^3?

(b) What is the volume of the trapezoidal section in ft^3?

(c) What is the total volume of the grit chamber in ft^3?

(d) At a flow of 6.3 MGD., what is the rate of flow in $\frac{ft^3}{sec}$?

(e) What is the detention time of the grit chamber at this flow rate?

CLARIFIERS

The most important purpose of the clarifier is to permit solids to settle to the bottom, where they are collected and pumped to another unit where they receive further treatment or are prepared for disposal. The fact that grease and scum are also removed from the surface is a fringe benefit. Mathematically, we are concerned with the factors which are directly related to solids settling.

For effective solids removal, we must allow sufficient time for settling, "Detention Time."

We must also provide enough area so that the majority of solids will have room to settle. If the solids can be spread over a greater area then better settling can be accomplished. The "Surface Settling Rate" is used as the measurement of the effect of area on settling.

The effluent weirs are placed in the clarifier to permit effluent to flow out of the tank in a clear thin layer off the surface of the settling sewage. If the flow over the weirs is too great, the resulting currents will carry solids up from the lower depths of the clarifier and out in the effluent. For this reason the "Weir Overflow Rate" must be kept within certain limits.

The usual optimum ranges for these measurements are:

Detention Time --- 2 - 4 hours

Surface Settling Rate --- 300 - 800 gal/ ft^2/ day

Weir Overflow Rate --- 10,000 - 15,000 gal/ linear ft/ day

Detention Time

$$\text{Detention Time (hrs)} = \frac{\text{Vol. of Clarifier (gal)}}{\text{Rate of Flow (gal/hr)}}$$

(*Factor the label: $DT = \frac{\text{gal}}{\frac{\text{gal}}{\text{hr}}} = \text{gal} \times \frac{\text{hr}}{\text{gal}} = \text{hr}$)

Example

You have a rectangular clarifier which is 32 ft. wide and 80 ft. long. The sewage depth is 8 ft. What is the volume of the tank in gal?

If the influent flow to this clarifier is 2.0 MGD, what is the rate of flow in gal/hr? What is the detention time?

Step 1 Volume of Clarifier (gal) = L(ft) x W(ft) x h(ft) x 7.48 (gal/ft^3)

(*Factor the label: Vol. = $ft \times ft \times ft \times \frac{gal}{ft^3} = gal$)

Volume of Clarifier = 80 ft x 32 ft x 8 ft x 7.48 gal/ft^3

= 20,480 ft^3 x 7.48 gal/ft^3

= 153,190 gal

Step 2 Rate of Flow = $\frac{\text{Flow (MGD)} \times 1{,}000{,}000 \text{ gal/MG}}{24 \text{ hrs/day}}$

(Factor the label: Rate of flow $= \frac{\frac{MG}{day} \times \frac{gal}{MG}}{\frac{hrs}{day}}$

$= \frac{MG}{day} \times \frac{gal}{MG} \times \frac{day}{hrs} = \frac{gal}{hr}$)

Rate of Flow = $\frac{2.0 \text{ MGD} \times 1{,}000{,}000 \text{ gal/MG}}{24 \text{ hrs/day}}$

= $\frac{2{,}000{,}000}{24}$ gal/hr

= 83,333 gal/hr

Step 3 Detention Time = $\frac{\text{Vol of Clarifier (gal)}}{\text{Rate of Flow (gal/hr)}}$

= $\frac{153{,}190 \text{ gal}}{83{,}333 \text{ gal/hr}}$

= 1.84 hrs

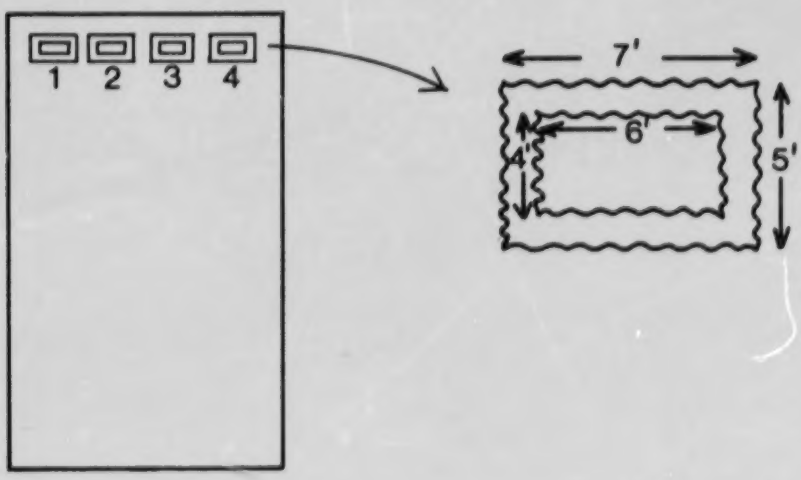
Weir Overflow Rate

Weir Overflow Rate (gal/ft/day) = $\frac{\text{Rate of Flow (gal/day)}}{\text{Length of Weir (ft)}}$

(Factor the label WOR = $\frac{\text{gal/day}}{\text{ft}}$ = gal/ft/day)

Example

The plant described with a flow of 2.0 MGD has four rectangular weirs with the following measurements:



Step 1. The sewage flows over the inside and outside weirs. What is the total length of the weir?

Length of One Weir = Outside Weir (1) + Inside Weir (1)
(L + W) x 2 (L + W) x 2

= (7ft + 5ft) x 2 + (6ft + 4ft) x 2

= 12ft x 2 + 10 ft x 2

= 24 ft + 20 ft = 44 ft

Total Length of Weir = Length 1 Weir ft. x 4

= 44 ft x 4

= 176 ft

Step 2. Rate of Flow = Flow (MGD) x 1,000,000 gal/MG

(Factor the label: Rate of Flow = $\frac{MG}{D} \times \frac{gal}{MG}$ = gal)

= 2.0 MGD x 1,000,000 gal/MG

= 2,000,000 gal/day

Step 3. Weir Overflow Rate = $\frac{\text{Rate of Flow gal/day}}{\text{Length of Weir (ft)}}$

Weir Overflow Rate = $\frac{2,000,000 \text{ gal/day}}{176 \text{ ft}}$

Weir Overflow Rate = 11,364 gal/ft/day

Surface Settling Rate

Surface Settling Rate = $\frac{\text{Rate of Flow gal/day}}{\text{Surface Area (ft}^2\text{)}}$

(Factor the label: SSR = $\frac{\text{gal/day}}{\text{ft}^2}$ = gal/ft^{2}day)

Example

Using the flow of 2.0 MGD and the dimensions of the clarifier of 32 ft. wide and 80 ft. long, determine the surface settling rate.

Step 1. . Rate of Flow = 2 MGD x 1,000,000 gal/MG

= 2,000,000 gal/day

Step 2. Surface Area = L (ft) x W (ft)

= 80 ft x 32 ft

= 2560 ft^2

Step 3. Surface Settling Rate = $\frac{\text{Rate of Flow (gal/day)}}{\text{Surface Area (ft}^2\text{)}}$

= $\frac{2,000,000 \text{ gal/day}}{2560 \text{ ft}^2}$

= 781 gal/ft^2/day

Problem # 1 The Clarifier in your plant is 25 ft. in diameter and 10 ft. deep. The overflow weir is 24 ft. in diameter.

(a) What is the volume of the tank in ft^3?

(b) What is the volume of the tank in gal.?

(c) The average daily flow to this unit is 0.5 MGD. What is the flow in gal/hr?

(d) What is the detention time in the Clarifier?

(e) What is the length of the overflow weir?

(f) What is the Weir Overflow Rate?

(g) What is the surface area of this clarifier?

(h) What is the Surface Settling Rate?

ANAEROBIC DIGESTERS

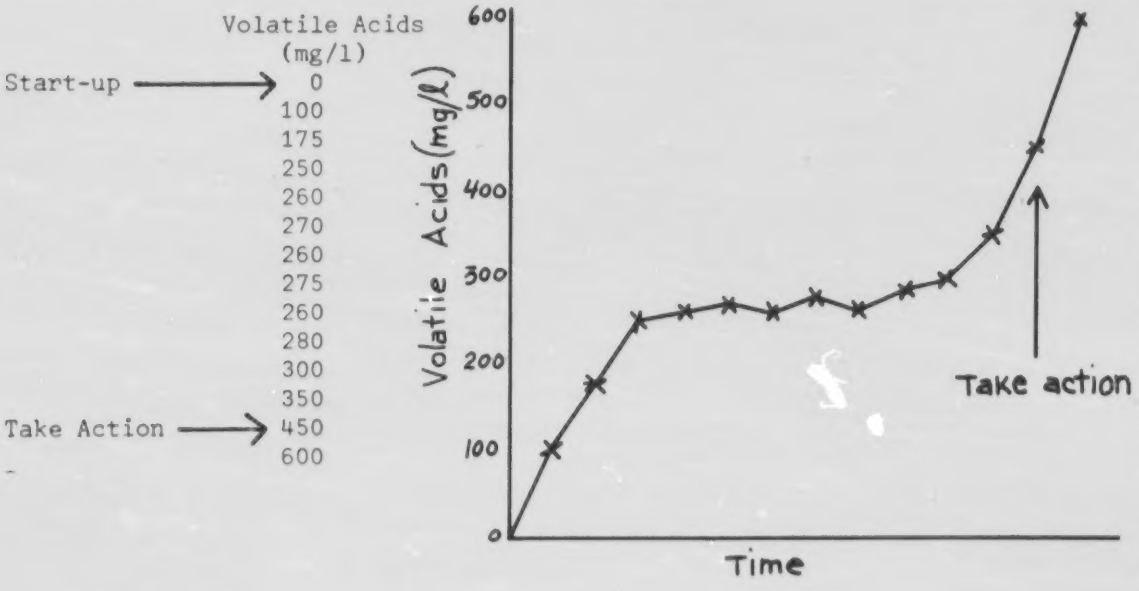
There are two principle types of bacteria with which we must be concerned for good digester control. They are the acid forming bacteria and the methane bacteria.

Initially, the acid forming bacteria break down the sludge particles into simpler compounds which dissolve in the water and can be absorbed by the bacteria. These simpler compounds are used as food by the bacteria for energy and new cell production. As a waste product of their digestion, they give off volatile acids and thus derive their name "acid forming bacteria." The methane bacteria in turn use the volatile acids as their food and give off methane and carbon dioxide wastes.

The methane bacteria are sensitive organisms and their growth and metabolism can be severely limited by excess volatile acids and an acid pH. If the acid forming bacteria produce volatile acids faster than the methane bacteria can convert them to methane and carbon dioxide, there will be a build up of volatile acids and a subsequent drop in pH. This in turn will further injure the methane bacteria and the level of volatile acids will build up at a more rapid rate and the digester will soon become acidic, resulting in failure. The drop in pH is usually slowed by the neutralizing power of the natural alkalinity in the sewage. The rapid development of low pH conditions can sometimes be slowed or stopped by the addition of alkalinity (lime etc.).

Generally the volatile acid level of well-operated anaerobic digesters will become fairly constant somewhere between 100 and 400 mg/l.

The volatile acid level should be plotted on a regular basis. A rise in the volatile acid level is the first sign of developing problems. Corrective action should be taken in the operation of the digester at the first clear sign of volatile acids build-up.

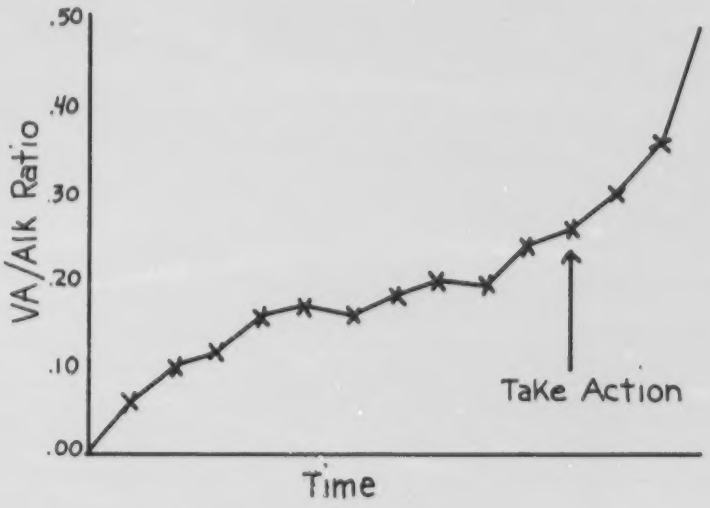


The neutralizing power of the alkalinity on the volatile acids and its effect on digester condition can be expressed as the Volatile Acids/Alkalinity Ratio.

Volatile Acids/Alkalinity Ratio	Digester Conditions
0.0 - 0.2	good operation
0.3 - 0.4	stress
0.5	deep trouble
0.8	failure

Corrective action in the operation of the digester should be taken at the first clear sign of a rise in the Volatile Acids/Alkalinity Ratio above 0.2.

	VA/Alk Ratio
Start up	.00
	.06
	.10
	.12
	.16
	.17
	.16
	.18
	.20
	.19
	.24
Take action	.26
	.30
	.36
	.50



Laboratory analyses of a digester sample give a volatile acids content of 240 mg/l and an alkalinity of 3000 mg/l. What is the VA/Alk Ratio?

$$\text{Vol Acids/Alk. Ratio} = \frac{\text{Volatile Acids (mg/l)}}{\text{Alkalinity (mg/l)}}$$

$$= \frac{240 \text{ mg/l}}{3{,}000 \text{ mg/l}}$$

$$= .08$$

DIGESTER LOADING

The organic solids, in the form of sludge, which are pumped to the digester, constitute the food for the anaerobic bacteria. These solids are defined as the volatile suspended solids (VSS). Underfeeding results in poor methane production and overfeeding can result in excessive volatile acids build up with subsequent digester upset.

A range of volatile solids, which can be fed to a digester and which will generally permit good operations, has been established. The range is based upon the number of pounds of volatile suspended solids, which can be utilized by a cubic foot of digester space occupied by the sludge (bacteria), and is called the digester loading.

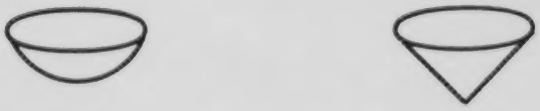
The digester loading is expressed as the pounds of volatile suspended solids per day which are fed to a cubic foot of digester occupied by the bacteria (sludge).

Type of Digester	Digester Loading (lbs VSS/ft^3/day)
Standard Rate (30 day D.T.)	0.03 - 0.10
High Rate (15 day D.T.)	0.10 - 0.40

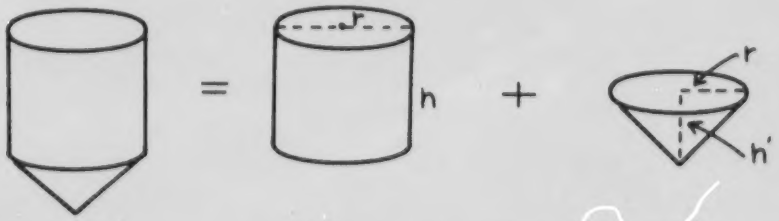
DIGESTER VOLUME

The volume of an anaerobic digester occupied by the sludge and bacteria may be expressed as the sum of the volume of a cylinder and the volume of the dish.

For process control, sufficient accuracy can be obtained by considering the dish as a cone rather than the segment of a sphere.



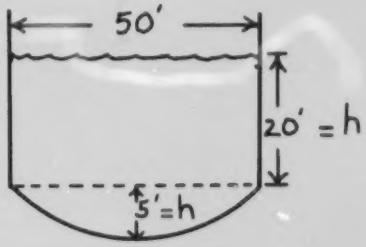
Thus, for practical considerations, the volume of the digester becomes the volume of the cylinder + the volume of the cone.



Vol. of Digester = $\pi r^2 h$ (Vol. of cylinder) + $\frac{1}{3} \pi r^2 h'$ (Vol. of cone)

Example #1. You pump 4400 lbs. of sludge/day to your digester. This raw sludge, upon laboratory analysis, is found to contain 75% volatile solids.

Your digester has a diameter of 50 ft. The sludge depth is 20 ft. above the top of the dish and the dish is 5 ft. deep.



What is the digester loading?

$$\text{Digester Loading} = \frac{\text{Volatile Suspended Solids (lbs/day)}}{\text{Vol. of Digester (ft}^3\text{) (used for sludge)}}$$

Step 1:

VSS (lbs) = Sludge pumped (lbs) x % volatile solids in Sludge

VSS = 4400 lbs x 0.75

VSS = 3300 lbs

Step 2:

Volume of digester = Vol. of cylinder (ft^3) + Vol. of dish (ft^3)

$$= \pi r^2 h + \frac{1}{3} \pi r^2 h'$$

$$= 3.14\ (25^2) \times 20 + \frac{1}{3} \times 3.14\ (25^2) \times 5$$

$$= 3.14 \times 625 \times 20 + \frac{1}{3} \times 3.14 \times 625 \times 5$$

$$= 3.14 \times 12500 + \frac{1}{3} \times 3.14 \times 3125$$

$$= 39250\ (ft^3) + 3271\ ft^3$$

$$= 42{,}521\ ft^3$$

Step 3:

$$\text{Digester Loading} = \frac{\text{VSS (lbs/day)}}{\text{Vol. of Digester (ft}^3\text{) used}}$$

$$= \frac{3300 \text{ lbs/day}}{42{,}521 \text{ ft}^3}$$

$$= 0.78 \text{ lbs/ft}^3\text{/day}$$

Example #2 The influent to your plant averages 4.0 MGD. The laboratory analysis of the influent gives a volatile suspended solids of 160 mg/l. If 95 percent of the suspended solids are removed in the primary clarifier, how many pounds of volatile suspended solids are collected and pumped to the digester/day?

Step #1

$$\text{Volatile Suspended Solids (lbs)} = \text{VSS Anal (mg/l)} \times 8.34 \times \text{Flow (MGD)}$$

$$= 160 \times 8.34 \times 4.0$$

$$= 640 \times 8.34$$

$$= 5338 \text{ lbs/day}$$

Step #2

$$\text{VSS Pumped} = \text{Influent VSS (lbs)} \times \% \text{ Removed in Primary}$$

$$= 5338 \text{ lbs} \times 0.95$$

$$= 5071 \text{ lbs/day}$$

If these solids are pumped to the digester in example #1, what is the digester loading?

$$\text{Digester Loading} = \frac{\text{V.S.S. (lbs/day)}}{\text{Vol. of Digester used (ft}^3\text{)}}$$

$$= \frac{5071 \text{ (lb/day)}}{42{,}521 \text{ ft}^3}$$

$$= 0.12 \text{ lbs/ft}^3\text{/day}$$

Problem #1

The sludge in your digester occupies 48,000 ft^3 of space. The The influent sewage contains 100 mg/l of volatile suspended solids at an average daily flow of 5.0 MGD. Assume that all of the solids settle out in the clarifier and are pumped to the digester.

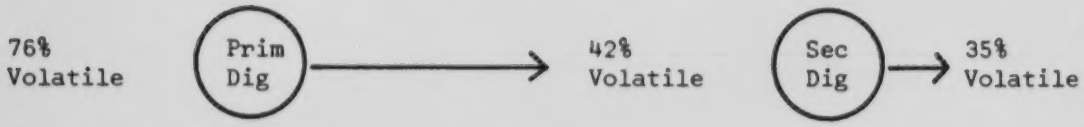
(a) How many pounds of volatile suspended solids are pumped to the digester?

(b) What is the digester loading?

Problem #2

A sample from the digester was analyzed inthe laboratory and was found to contain 210 mg/l of volatile acids and had an alkalinity of 1400 mg/l. What is the volatile acid/alkalinity ration?

Problem # 3: The raw solids being pumped to your primary anaerobic digester contain 76% volatile solids. The effluent solids from the primary digester contain 42% volatile solids and from the secondary digester 35% volatile solids.



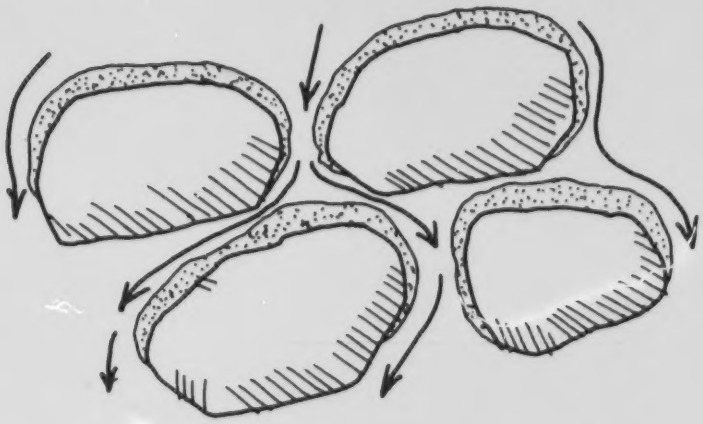
(a) What % of volatile solids are removed in the primary digester?

(b) What % of the volatile solids are removed in the secondary digester?

(c) What is the overall removal of volatile solids by anaerobic digestion?

TRICKLING FILTERS

The active microorganisms in the trickling filter are attached to a solid support media in the form of a slime coating over the surface of the media. Sufficient open spaces must be maintained down through the filter to allow the sewage to trickle down over the surface of the biological mass, thus supplying it with food. Oxygen for the biological digestion is obtained from the air flowing up and down through the filter. The oxygen dissolves in the sewage and then diffuses into the biological growth.



There are two factors which effect the efficiency of trickling filters to which we should give some mathematical consideration.

The first is the hydraulic loading or the volume of sewage which we allow to pass through and over this biological mass. Enough sewage must be fed to accomplish uniform distribution over the entire filter and to operate the sweep arms during the early stages of start-up operations. Insufficient

flow will often result in poor food distribution, insufficient slime growth, and poor removals. Excessive or high flows will also cause operating problems. Rapid flow through the filter reduces the contact time of the sewage with the active biological mass and thus may reduce the efficiency of removal. Excessive flows will also cause excessive washing off of the biological slime. In the case of excessive recycling or infiltration, the influent sewage may be too weak to provide sufficient food contact with the bacteria and low removals will occur. This is similar to watering the soup to the point where you can not eat enough soup to get sufficient nourishment.

The hydraulic loading of trickling filters is usually expressed as the volume of sewage applied to the surface of the filter per day, either as million gallons per acre per day (MGAD) or as gallons per square foot per day ($gal/ft^2/day$).

Generally, the desired hydraulic loadings to trickling filters fall within the following ranges.

Type of Filter	Hydraulic Loading (MGAD)	Hydraulic Loading ($gal/ft^2/day$)
Standard Rate	1.0 - 5.0	25 - 100
High Rate	5.0 -50.0	100 - 1000
Roughing	100	1000

Example: You have a trickling filter plant in which the average daily flow is 3.5 MGD. The trickling filter is 80 ft. in diameter. What is the hydraulic loading in $gal/ft.^2/day$ and in MGAD?

Step 1. Surface Area (ft^2) = $\pi \times r^2$

= 3.1416×40^2 ft^2

= 5027 ft^2

Surface Area (A) = $\frac{\text{Surface Area } (ft^2)}{43{,}560\ (ft^2/A)}$

= $\frac{5027}{43{,}560}$ A

= 0.115 A

Step 2. Hydraulic Loading ($gal/ft^2/day$) = $\frac{\text{Flow (gal/day)}}{\text{Surface Area } (ft^2)}$

= $\frac{3{,}500{,}000 \text{ gal/day}}{5027\ ft^2}$

= 696 $gal/ft^2/day$

Step 3. Hydraulic Loading MGAD = $\frac{\text{Flow (MGD)}}{\text{Surface Area (A)}}$

= $\frac{3.5 \text{ MGD}}{0.115 \text{ A}}$

= 30.4 MGAD

The second important consideration is the amount of food we supply to the active biological organisms. This value is called the organic or BOD loading.

The organisms within the trickling filter must be fed properly to efficiently digest the organic waste. Too little food (BOD) will result in poor biological growth and thus poor removal. High or excessive amounts of BOD will simply pass through the filter without being completely digested and again the BOD removal will be low.

For these reasons we should be able to determine the organic or BOD loading and keep it within ranges which will produce maximum efficiency.

To be a realistic value, the amount of food or BOD fed to a trickling filter must some how be related to the amount of active biological growth in the filter. In this case, it is assumed that a 1000 cubic feet of media will contain the same amount of biological mass. For this reason, the organic or BOD loading is expressed as the pounds of BOD applied to 1000 ft^3 of filter media per day.

Generally, the desirable BOD loadings fall within the following ranges.

Type of Filter	BOD Loading (lbs/1000 ft^3/day)
Standard Rate	5-25
High Rate	25-50
Roughing	100

Example: Your trickling filter which is 100 ft. in diameter contains 8 ft. of media. The flow to your plant averages 2.0 MGD and has an average BOD of 90 mg/l. What is the organic loading to the trickling filter?

$$\text{Organic Loading (BOD)} = \frac{\text{Influent BOD to T.F. (lbs/day)}}{\frac{\text{Volume Filter media } (ft^3)}{1{,}000}}$$

Step 1. Infl. BOD (lb/day) = BOD Anal. $(\cancel{mg}/l) \times 8.34 \left(\frac{lb/\cancel{MG}}{\cancel{mg}/l}\right) \times \text{Flow}\left(\frac{\cancel{MG}}{day}\right)$

= 90 x 8.34 x 2.0 lb/day

= 1501.2 lb day

Step 2. Volume Media (1000 ft^3) $= \frac{\pi \times r^2 \times h}{1000}$

$= \frac{3.1416 \times 50^2 \times 8 \text{ ft}^3}{1000}$

$= \frac{62,832 \text{ ft}^3}{1,000}$

$= 62.832 \text{ (1000 ft}^3\text{)}$

Step 3. Organic Loading (lb BOD/1000 ft^3/day)

$= \frac{\text{BOD Loading (lbs/day)}}{\text{Vol. Filter Media (1000 ft}^3\text{)}}$

$= \frac{1501.2 \text{ lb BOD/day}}{62.832 \text{ (1000 ft}^3\text{)}}$

$= 23.9$ lbs BOD/1000 ft^3/day

Porblem #1

Your trickling filter is 95 feet in diameter and the flow to it is 5.0 MGD.

(a) What is the surface area of the tank in ft^2?

(b) What is the hydraulic loading (gal/ft^2/day)?

Problem #2

Your trickling filter is 90 feet in diameter, the depth of the media is 8.5 feet, the influent B.O.D. is 180 mg/1 and the flow is 4.5 MGD.

(a) What is the volume of the media in ft^3?

(b) What is the volume of the media in 1000 ft^3?

(c) What is the B.O.D. loading per 1000 ft^3/day?

ACTIVATED SLUDGE UNITS

The best removal of organic material in an activated sludge plant depends upon maintaining a constant level of food in proportion to the number of organisms present in the system. The number of organisms is measured by the mixed liquor suspended solids (MLSS) in the aeration unit.

Since the volume and strength of the sewage entering the treatment facility must be accepted at all times, the "food" supply cannot be changed. The ratio of food to organisms (Food/Mass Ratio) is, therefore, controlled by the amount of activated sludge (biological organisms) wasted or recycled to the system.

The level of organisms present (MLSS) usually varies between 1,000 and 4,000 mg/l.

The ratio of food to organisms (F/M Ratio) in aeration units may vary from 0.2 to 0.5 depending upon the type of process and the operating conditions. Generally, loadings of 30 to 35 lbs. of BOD per 100 lbs. of activated sludge are good operating levels. These loadings represent F/M Ratios of 0.3 to 0.35 respectively.

It is also necessary for incoming solids to remain in contact with the aerobic organisms long enough for digestion to take place. Normally a sludge age of from 3 to 7 days in the aeration unit is sufficient for expected digestion to take place.

Aerobic digesters should be sufficiently large to insure a sludge age of at least 15 days if odors are to be avoided upon drying the sludge.

A major concern of activated sludge processing is developing a good settling sludge. Frequent testing and graphing of the settling rate is an excellent means of detecting changes in sludge characteristics.

The classical test for measuring sludge settling has been the Sludge Volume Index (SVI). A desirable Sludge Volume Index will vary with the type of aeration. The best Sludge Volume Index is usually somewhat lower for diffused air system than for mechanical air systems. There may be considerable variation from plant to plant. Maximum values can only be determined with constant use and observation. Generally good settling sludge will exhibit a SVI of less than 200. Sludge Volume Indices which are steadily increasing toward the 300-400 range indicate an impending bulking situation.

WEIGHT OF SOLIDS or BOD

One gallon of water weighs 8.34 lbs.

In sewage treatment, we also make the assumption that suspended solids and BOD have the same weight as water. It is this assumption that enables us to use this factor to calculate the pounds of suspended solids or BOD entering a plant or treatment unit as follows:

1 ppm = 1 mg/l = 1 gal/1,000,000 gal

1 gal = 8.34 lbs

if we substitute 8.34 lbs for a gallon
then 1 ppm = 1 mg/l = 8.34 lbs/MG

this means that for each MG of influent 1 mg/l of suspended solids or BOD can be expressed as 8.34 lbs

if there is an influent of 10 MG, then the weight of solids or BOD will be

$$1 \text{ mg/l} = 8.34 \frac{\text{lbs}}{\cancel{\text{MG}}} \times 10 \cancel{\text{MG}} = 83.4 \text{ lbs}$$

Influent BOD = 160 mg/l

$$\text{weight of BOD (lbs)} = 160 \cancel{\text{mg/l}} \times 83.4 \frac{\text{lbs}}{\cancel{\text{mg/l}}}$$

$$= 13{,}280 \text{ lbs}$$

$\therefore$ Formula BOD (lbs) = Influent BOD ($\cancel{\text{mg/l}}$) $\times \frac{8.34 \text{ lbs}/\cancel{\text{MG}}}{\cancel{\text{mg/l}}} \times$ Flow ($\cancel{\text{MG}}$)

Formula SS (lbs) = Influent SS ($\cancel{\text{mg/l}}$) $\times \frac{8.34 \text{ lbs}/\cancel{\text{MG}}}{\cancel{\text{mg/l}}} \times$ Flow ($\cancel{\text{MG}}$)

$$\underline{\text{Food/Mass (F/M) Ratio}} = \frac{\text{Influent BOD (lbs)}}{\text{Aeration unit MLSS (lbs)}}$$

Example: Aeration Tank = 180 ft. long
60 ft. wide
14.3 ft. deep

MLSS = 2400 mg/l

Flow = 7.0 MGD

Influent BOD = 120 mg/l
SS = 105 mg/l

Determine the Food/Mass Ratio.

$$\underline{\text{Influent BOD (lbs)}} = \text{Infl. BOD (mg/l)} \times \frac{8.34 \text{ lb/MG}}{\text{mg/l}} \times \text{Flow (MG)}$$

$$= 120 \times 8.34 \times 7.0 \text{ lbs}$$

$$= 840 \times 8.34 \text{ lbs}$$

$$= 7006 \text{ lbs}$$

$$\underline{\text{Vol. Aeration Unit (gal)}} = 180 \text{ ft} \times 60 \text{ ft} \times 14.3 \text{ ft} \times 7.48 \text{ gal/ft}^3$$

$$= 10800 \times 14.3 \times 7.48 \text{ gal}$$

$$= 154{,}440 \times 7.48 \text{ gal}$$

$$= 1{,}155{,}211 \text{ or } 1{,}155{,}000 \text{ gal}$$

$$= 1.155 \text{ MG}$$

$$\underline{\text{MLSS in Aer. Unit (lbs)}} = \text{MLSS (mg/l)} \times 8.34 \frac{\text{(lb/MG)}}{\text{mg/l}} \times \text{Vol. Tank (MG)}$$

$$= 2400 \times 8.34 \times 1.155 \text{ lbs}$$

$$= 20016 \times 1.155 \text{ lbs}$$

$$= 23{,}118 \text{ lbs}$$

$$\underline{\text{F/M Ratio}} = \frac{\text{Influent BOD (lbs)}}{\text{MLSS in Aer. Unit (lbs)}}$$

$$= \frac{7006 \text{ lbs}}{23118 \text{ lbs}}$$

$$= 0.303$$

$$\text{Sludge Age (days)} = \frac{\text{Suspended Solid in Aeration Unit (lbs)}}{\text{Suspended Solids in Influent (lbs/day)}}$$

$$\text{S.S. in Aeration Unit} = \text{MLSS (mg/l)} \times 8.34 \left(\frac{\text{lb/MG}}{\text{mg/l}}\right) \times \text{Vol. Tnk (MG)}$$

$$= 2400 \times 8.34 \times 1.155 \text{ lbs}$$

$$= 20016 \times 1.155 \text{ lbs}$$

$$= 23{,}118 \text{ lbs}$$

$$\text{SS in Influent (lbs/day)} = \text{Inf. SS (mg/l)} \times 8.34 \left(\frac{\text{lb/MG}}{\text{mg/l}}\right) \times \text{Flow (MGD)}$$

$$= 105 \times 8.34 \times 7.0 \text{ lbs/day}$$

$$= 735 \times 8.34 \text{ lbs/day}$$

$$= 6130 \text{ lbs/day}$$

$$\text{Sludge Age (days)} = \frac{\text{SS in Aeration Unit (lbs)}}{\text{SS in Influent (lbs/day)}}$$

$$= \frac{23{,}118 \text{ lbs}}{6{,}130 \text{ lbs/day}}$$

$$= 3.77 \text{ days}$$

$$\text{Sludge Volume Index (SVI)} = \frac{\text{*Volume Settled Sludge (ml/l) x 1000}}{\text{Suspended Solids (mg/l)}}$$

Problem: An activated sludge has a settled volume of 290 ml after 30 minutes. If the sample contains 2400 mg/l of suspended solids what is the SVI?

$$\text{SVI} = \frac{\text{Vol. Settled Sludge (ml/l) x 1000}}{\text{Suspended Solids (mg/l)}}$$

$$= \frac{290 \times 1000}{2400}$$

$$= \frac{290{,}000}{2400}$$

$$= 121$$

*Volume of settled sludge is measured after 30 minutes settling in a 1000 ml. graduated cylinder.

ACTIVATED SLUDGE PROBLEMS

Problem #1

Your aeration tank is 200 feet long, 50 feet wide, and is filled to a depth of 10 feet with activated sludge. The flow to the plant is 5.0 MGD. Influent samples show an influent BOD of 105 mg/1 and influent suspended solids of 150 mg/1. The mixed liquors are analyzed and found to contain a MLSS of 3000 mg/1.

(a) What is the volume of the tank in ft^3?

(b) What is the volume of the tank in gal?

(c) How many pounds of BOD are entering the plant?

(d) How many pounds of suspended solids are in the influent?

(e) How many pounds of MLSS are in the aeration tank?

(f) What is the Food/Mass (F/M) Ratio?

(g) What is the sludge age in days?

Piontkowski's Point

Why is it?

There is never enough time to do it right?

Always enough time to do it over.

SLUDGE HANDLING

There are two major factors we must consider in the separation of solids and the disposal of the sludge cake.

First of all, we want to remove as many solids as we can from the process streams either by centrifuging or filtering. Any solids which pass through the centrifuge or filter and return to the head of the plant in the return liquors must be processed over again. For this reason, we must be concerned with the percent capture of solids.

In general, the percent capture by centrifuging ranges from 60--75 percent on primary digested solids to a low of 50 - 65 percent on primary plus waste activated sludges. Polymers are often used to raise these figures to the 80 - 95 percent level.

$$\% \text{ Capture} = \frac{\text{SS Sludge slurry (mg/l)} - \text{SS return liquor (mg/l)}}{\text{SS sludge slurry (mg/l)}} \times 100$$

Example:

Sludge to be centrifuged = 12,500 mg/l

Return liquors = 4,500 mg/l

Find the % Capture.

$$\% \text{ Capture} = \frac{12{,}500 \text{ (mg/l)} - 4{,}500 \text{ (mg/l)}}{12{,}500 \text{ (mg/l)}} \times 100$$

$$\% \text{ Capture} = \frac{8{,}000}{12{,}500} \times 100$$

$$\% \text{ Capture} = 64$$

It is desirable to produce as dry a filter or centrifuge cake as possible. Lower moisture in the sludge cake will result in lower hauling costs, less drying time, and lower combustion costs if incinerated. It is, therefore, important to know the amount of moisture in the disposable sludge.

$$\% \text{ Moisture} = \frac{\text{wgt sample (gms)} - \text{wgt dry solids (gms)}}{\text{wgt sample (gms)}}$$

Example: A 50 gm sample of centrifuge cake was dried in the laboratory at 103-105°C. The dry solids weighed 15.8 gms. What was the percent of moisture in the sludge?

$$\% \text{ Moisture} = \frac{\text{wgt sample (gms)} - \text{wgt dry solids (gms)}}{\text{wgt sample (gms)}} \times 100$$

$$= \frac{50-15.8}{50} \times 100$$

$$= \frac{34.2}{50} \times 100$$

$$= 68.4$$

Truesdells' Truism

The worker who reads instructions before starting a job

Gets knowledge.

The worker who does not.

Gets experience.

Problem #1

A 10.5 percent slurry of digested sludge is pumped to your centrifuge. The centrate returned to the head of the plant contains 47,250 mg/l of suspended solids.

(a) What is the suspended solids content of the feed in mg/l? (Hint - Change the percent to a decimal and multiply by 1,000,000.)

(b) What is the percent solids removed by centrifuging?

Problem #2

An evaporating dish weighing 68.2 gms was filled with 50 gms of sludge cake from your vacuum filters. After being dried in the laboratory the dish and the dry sample weighed 79.3 gms. What is the percent of moisture in the filter cake?

CHLORINATION CHAMBERS

A requirement of all sewage treatment is the disinfection of the plant effluent. The purpose of disinfection is to destroy pathogenic or disease causing organisms. Chlorination has become the most commonly used method because it is effective and economical.

There are two theories of how chlorine kills or disinfects. One is that the chlorine attacks the bacteria cell directly and destroys it. The other theory is that the chlorine destroys the bacterial external enzymes and the organisms die of starvation. Regardless of the mechanism there are two facts which are important to us and for which we will need to use our math. First, it takes time to kill these micro-organisms. Secondly, like all toxic substances, chlorine takes longer to kill in weak concentrations than in strong concentrations.

Thus, we are concerned with two factors, detention time which is the contact time necessary to kill the disease organisms and the chlorine residual or the effective killing concentration of the chlorine. The generally accepted minimum standard is a chlorine residual of 0.5 mg/l for 15 minutes.

Chlorine is a very active chemical and reacts with many substances in sewage. Some of these substances such as hydrogen sulfide and ferrous iron react with the chlorine and, in the process, destroys its ability to kill bacteria. These substances usually react the fastest with the chlorine and so they use up or destroy some chlorine before any disinfection can occur. The amount of chlorine used up in reacting with these substances is called "Chlorine Demand."

The chlorine in the sewage in excess of this demand has the ability to kill micro-organisms and is called the "Chlorine Residual." It is our intention, therefore, to add sufficient chlorine "Chlorine Dose" so that we surpass the "Chlorine Demand" and have enough left over "Chlorine Residual" to kill all of the disease causing organisms in the sewage before it is discharged to our lakes and streams.

From this information we have the chlorination formula:

$$\text{Dose} = \text{Demand} + \text{Residual}$$

Like so many of our measurements the chlorine concentrations are expressed in terms of milligrams per liter (mg/l). Thus our formula becomes:

$$\text{Dose (mg/l)} = \text{Demand (mg/l)} + \text{Residual (mg/l)}$$

If we wish to maintain the minimum acceptable chlorine residual of 0.5 mg/l the dosage we must use will be 0.5 mg/l greater than the demand.

$$\text{Dose (mg/l)} = \text{Demand (mg/l)} + 0.5 \text{ (mg/l)}$$

Chlorine demand can be measured in the laboratory by testing a sample of sewage. However, the test takes some time, and the chlorine demand of the sewage often changes so rapidly that the calculated dosage no longer applies to the sewage leaving the plant at the end of the test. Because of the rapidly changing characteristics of sewage and the relative complexity of the laboratory tests, repeated manual testing of chlorine demand has been largely abandoned.

For those plants which do not have facilities for continuous instrumental analysis and control of chlorine addition or at times when these instruments are out of service, chlorine control must be exercised manually.

The usual procedure is to measure the chlorine demand during the peak hours, usually between 10 A.M. and 2 P.M., and to add chlorine at such a rate as to maintain a residual of 0.5 mg/l during that period. As the flow and demand drops off during the day and night, the residual increases and disinfection is maintained.

To illustrate:

TIME	CHLORINE DOSE (mg/l)	CHLORINE DEMAND (mg/l)	CHLORINE RESIDUAL (mg/l)
10 A.M.	10.0	9.5	0.5
2 P.M.	10.0	9.0	1.0
6 P.M.	10.0	8.5	1.5
10 P.M.	10.0	8.2	1.8
2 A.M.	10.0	8.0	2.0
6 A.M.	10.0	8.0	2.0

This procedure assures adequate chlorine residual for disinfection but results in the waste of chlorine. It may also have a detrimental effect upon marine life of the receiving waters but we should not lose sight of the fact that the primary purpose of chlorination is to prevent disease. If the above case was typical of daily operations, then this problem could be reduced by cutting back the chlorine dosage to 9.0 mg/l from 4 P.M. to 8 A.M.

Chlorine dosage is measured in mg/l. Chlorine usage in the plant is measured in terms of pounds per day (lbs/day) and is purchased by the pound. If a constant feed rate is maintained as in the above illustration the amount of chlorine needed can be calculated as follows:

$$\text{Rate of Feed } \left(\frac{\text{lbs}}{\text{day}}\right) = \text{Dose } (\cancel{\text{mg/l}}) \times 8.34 \left(\frac{\text{lbs/}\cancel{\text{MG}}}{\cancel{\text{(mg/l)}}}\right) \times \text{Flow } (\cancel{\text{MG/D}})$$

Example #1

Laboratory analyses showed that during the peak flow period the chlorine demand reached a high of 7.0 mg/l. If the chlorine dosage was adjusted to give a chlorine residual of 0.5 mg/l what should the dosage be? If the flow in the plant averages 1.2 MGD how many lbs/day of chlorine will be needed for disinfection?

Step #1 Dose (mg/l) = Demand (mg/l) + Residual (mg/l)

= 7.0 (mg/l) + 0.5 mg/l

= 7.5 mg/l

Step #2 Rate of Feed (lbs/day) = Dose $(\cancel{mg/l}) \times 8.34 \frac{lbs/\cancel{MG}}{\cancel{mg/l}} \times$ Flow $\cancel{MG}/D$

= 7.5 x 8.34 x 1.2 lbs/day

= 75.1 lbs/day

The detention time of the chlorination chamber is intended to be sufficient to provide time for the chlorine to destroy the pathogenic organisms. In the case of a minimum chlorine residual of 0.5 mg/l this should be at least 15 minutes.

Detention time is calculated using the usual formula.

$$\text{Detention Time} = \frac{\text{Volume of Chlorine Contact Tank (gal)}}{\text{Flow (gal/min)}}$$

$$\text{Detention Time (min)} = \frac{\text{Volume of Chlorine Tank (gal)}}{\text{Flow } (\frac{\text{gal}}{\text{day}})}$$

$$\frac{}{24 \frac{\text{hrs}}{\text{day}} \times 60 \frac{\text{min}}{\text{hr}}}$$

$$= \frac{\text{Vol. Tank (gal)} \times 24 \ (\frac{\cancel{\text{hrs}}}{\text{day}}) \times 60 \ (\frac{\text{min}}{\cancel{\text{hr}}})}{\text{Flow } (\frac{\text{gal}}{\text{day}})}$$

Factor the Label

$$\left((\text{min}) = \frac{\text{gal} \times \frac{\text{hr}}{\text{day}} \times \frac{\text{min}}{\text{hr}}}{\frac{\text{gal}}{\text{day}}} \right)$$

$$= \text{gal} \times \frac{1}{\text{day}} \times \text{min} \times \frac{\text{day}}{\text{gal}}$$

$$\text{min} = \text{min} \quad)$$

Example #2

Your chlorine contact chamber is 5 ft. wide, 6 ft. deep, and 24 ft. long. At a flow of 0.5 MGD what is the detention time in the chlorination chamber?

Step #1 Volume of Chlorine Tank (ft^3) = L (ft) x w (ft) x D (ft)

= 24 ft x 5 ft x 6 ft

= 720 ft^3

Step #2 Volume of Chlorine Tank (gal) = Vol. of tnk (ft^3) x 7.48 (gal/ft^3)

= 720 x 7.48 gal.

= 5385.6 gal

Step #3 Detention Time (min)

$$= \frac{(\text{gal}) \times 24 \frac{\text{hr}}{\text{day}} \times 60 \frac{\text{min}}{\text{hr}}}{\text{Vol. Flow (gal/day)}}$$

$$= \frac{5385.6 \times 24 \times 60}{500{,}000} \text{ min}$$

$$= \frac{7{,}755{,}264}{500{,}000} \text{ min}$$

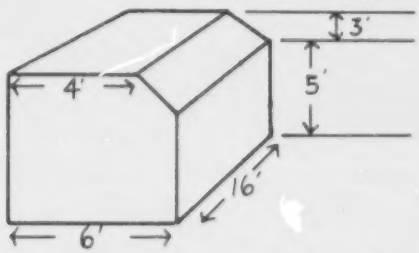
= 15.5 min

Remember the contact time (detention time) in the chlorination chamber can be severely shortened by high infiltration during wet weather or by short circuiting in the treatment unit. For these reasons it is advisable to measure the detention time by dye testing as well as by calculation. Increased chlorine residual may be necessary for effective disinfection when the detention time is short.

PROCESSING REVIEW PROBLEMS

1. The Grit Chamber in your plant is 15 feet long. Upon dye testing it was found that the dye passed through the chamber in 20 seconds. What is the flow rate in the chamber?

2. Your grit chamber has the following dimensions.



a. What is the volume of the trapezoidal section in $ft.^3$?

b. What is the volume of the rectangular section in $ft.^3$?

c. What is the volume of this grit chamber in $ft.^3$?

d. What is the volume in gallons?

3. The weirs on the effluent trough of your circular clarifier have an inside diameter of 35 ft. and an outside diameter of 36 ft.

   a. What is the length of the inside weir?

   b. What is the length of the outside weir?

   c. What is the total weir length?

   d. When the flow through the plant is 1.7 MGD, what is the Weir Overflow Rate?

4. Laboratory analyses of the sludge from your anaerobic digester gave the following results:

   pH = 6.8, alkalinity = 2300 mg/l, volatile acids = 184 mg/l

   What is the volatile acids/alkalinity ratio?

5. Your trickling filter contains 38,500 ft^3 of media. The flow to your plant is 0.5 MGD. The primary effluent contains 110 mg/l of BOD.

   a. How many pounds of BOD are applied to the trickling filter per day?

   b. What is the organic or BOD loading?

6. The sludge being fed to your centrifuge contains 7.5 percent suspended solids.

   a. What is the suspended solids content in mg/l?

   b. If the centrate returned to the wet well contains 15,000 mg/l of suspended solids what percent of the solids are removed by the centrifuge?

7. The chlorine demand of your effluent is 3.5 mg/l. The chlorine residual must be maintained at 0.5 mg/l.

   a. What dosage rate should be applied to the effluent?

   b. If the average flow through the plant is 6.0 MGD, how many pounds of chlorine are used daily to maintain the desired residual?

8. A 40 ml sample of sewage was filtered through a gooch funnel, dried in the drying oven, cooled and weighed. If the gooch weighed 18.0268 gms and the gooch plus the dry solids weighed 18.0330 gms (a) What is the suspended solids analysis of the sewage?

9. The dry solids from 50 gms of sludge weighed 7.2 gms. After ignition the sample weighed 2.2 gms.

   a. What is the percent of dry solids in the sample?

   b. What percent of the solids are volatile?

Linberg's "Principle of the Vital Few"

90% of the work is accomplished by 10% of the workforce.

BASIC UNITS

English System

Linear	1 foot (ft)	=	12 inches (in)
	1 yard (yd)	=	36 inches (in)
	1 yard (yd)	=	3 feet (ft)
	1 mile (mi)	=	5280 feet (ft)
	1 mile (mi)	=	1760 yards (yds)
Area	1 square foot (ft^2)	=	144 square inches (in^2)
	1 square yard (yd^2)	=	9 square feet (ft^2)
	1 acre (A)	=	43,560 square feet (ft^2)
Volume	1 cubic foot (ft^3)	=	1728 cubic inches (in^3)
	1 cubic yard (yd^2)	=	27 cubic feet (ft^3)
	1 cubic foot (ft^3)	=	7.48 gallons (gal)
	1 gallon (gal)	=	4 quarts (qts)
Weight	1 pound (lb)	=	16 ounces (oz)
Water	1 cubic foot water	=	62.4 pounds (lbs)
	1 gallon water	=	8.34 pounds (lbs)

1 part per million (ppm) = 1 gallon per million gallons (1 gal/MG)

1 part per million water = 8.34 lbs. per million gallons (lbs/MG)

"Think Trout"

Rich

BASIC UNITS

Metric System

Linear	1 centimeter (cm)	=	10 millimeters (mm)
	1 meter (m)	=	100 centimeters (cm)
	1 kilometer (km)	=	1000 meters (m)
Area	1 square meter (m^2)	=	10,000 square centimeters (cm^2)
Volume	1 milliliter (ml)	=	1 cubic centimeter (cc)
	1 milliliter (ml)		
	or		0.001 liter (l)
	1 cubic centimeter (cc)		0.001 liter (l)
	1 liter (l)	=	1000 milliliters (ml)
			or
			1000 cubic centimeters (cc)
	1 kiloliter (kl)	=	1000 liters (l)
Weight	1 milligram (mg)	=	0.001 gram (gm)
	1 gram (gm)	=	1000 milligrams (mg)
	1 kilogram (kg)	=	1000 grams (gm)

1 part per million (ppm) = 1 milligram per kilogram (mg/kg)

1 part per million water = 1 milligram per liter (mg/l)
1 part per million water = 1 gram per cubic meter (g/m^3)

CONVERSION TABLES

English System to Metric System

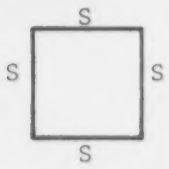
English Unit (Symbol)		Multiply by	to get Metric Unit	(Symbol)
Acre	(A)	0.405	Hectares	(ha)
Cubic foot	(ft^3)	0.0283	Cubic meters	(m^3)
Cubic foot	(ft^3)	28.32	Liters	(l)
Cubic feet/minute	(cfm)	0.472	Liter/second	(l/sec)
Cubic feet/second	(cfs)	0.0283	Cubic meters/second	(m^3/sec)
Cubic yard	(yd^3)	0.765	Cubic meter	(m^3)
Foot	(ft)	0.305	Meter	(m)
Feet/second	(fps)	30.5	Centimeter/second	(cm/sec)
Gallon U.S.	(gal)	3.785	Liters	(l)
Gallons/linear foot/day (gal/ft/day)		0.0124	Cubic meter/meter/day	(m^3/m/day)
Gallons/square foot/day (gal/ft^2/day)		0.0407	Cubic meter/square meter/day	(m^3/m^2 day)
Gallons/minute	(gpm)	0.0631	Liters/second	(l/sec)
Inches	(in)	25.4	Millimeters	(mm)
Million gallons (MG)		3785	Cubic meters	(m^3)
Million gallons/day (MGD)		43.81	Liter/second	(l/sec)
Million gallons/day (MGD)		0.0438	Cubic meters/second	(m^3/sec)
Million Gallons/acre/day (MGAD)		9350	Cubic meters/hectare/day	(m^3/ha/day)
Pounds (mass)	(lb)	0.454	Kilograms	(kg)
Pounds/cubic foot	(lbs/ft^3)	16.02	Kilograms/cubic meter	(kg/m^3)
Pounds/1000 cubic feet/day (lb/1000 ft^3/day)		0.016	Kilograms/cubic meter/day	(kg/m^3/day)
Pounds/acre/day	(lbs/A/day)	1.121	Kilograms/hectare	(kg/ha)
Square foot	(ft^2)	0.0929	Square meter	(m^2)
Ton (short)	(ton)	0.907	Ton (metric)	(t)
Yard	(yd)	0.914	Meters	(m)
Part per million	(ppm)	1.000	Milligrams/liter	(mg/l)
"	"	1.000	Gram/cubic meter	(g/m^3)

DIMENSIONAL FORMULAS

Perimeters, Areas, and Volumes
(pages 60-74)

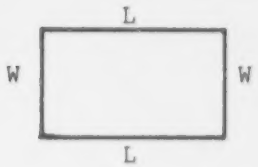
Perimeters

Square:



$P = S + S + S + S = 45$

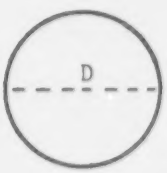
Rectangle:



$P = L + W + L + W = 2L + 2W$

Circle:

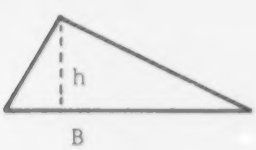
$C = 2\pi r$

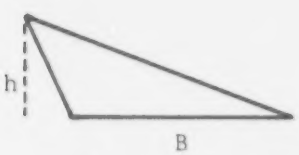


$C = \pi D$

Areas:

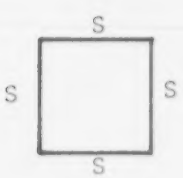
Triangle:





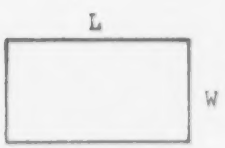
$A = \frac{1}{2} B \times h$

Square:



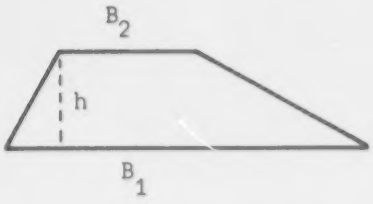
$A = S^2$

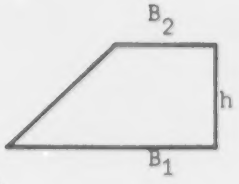
Rectangle:



Areas continued:

Trapezoid:

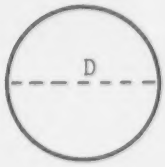




$$A = \left(\frac{B_1 + B_2}{2}\right) h$$

Circle:

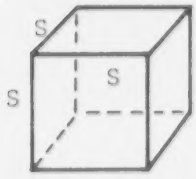
$$A = \pi r^2$$



$$A = \pi \frac{D^2}{4}$$

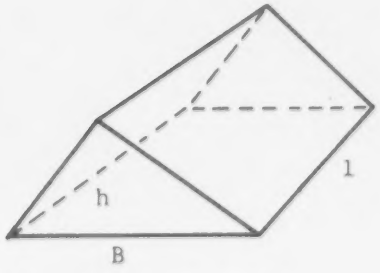
Volumes:

Cube:



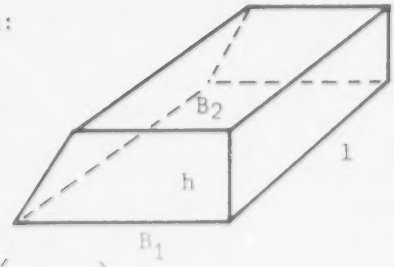
$$V = S \times S \times S = S^3$$

Triangular Section:



$$V = \frac{1}{2} B \times h \times l = A \times l$$

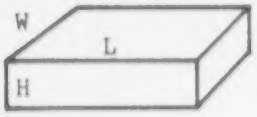
Trapezoidal Section:



$$V = \left(\frac{B_1 + B_2}{2}\right) \times h \times l = A \times l$$

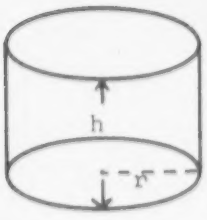
Volumes continued:

Rectangular Tank:



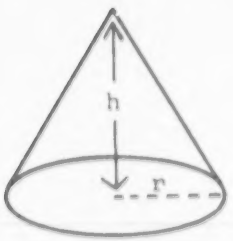
$V = L \times W \times H$

Cylinder:



$V = \pi r^2 h = \pi \frac{D^2}{4} h$

Cone:



$V = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi \frac{D^2}{4} h$

PROCESSING FORMULAS

Grit Chambers (pages 127-137)

$$\text{Flow Velocity (ft/sec)} = \frac{\text{Length of Grit Chamber (ft)}}{\text{Time for sewage to pass through G.C. (sec)}}$$

$$\text{Flow Velocity (ft/sec)} = \frac{\text{Rate of Flow to Grit Chamber (ft}^3\text{/sec)}}{\text{Cross sectional area of sewage in the channel (ft}^2\text{)}}$$

$$\text{Detention Time (sec)} = \frac{\text{Volume of Grit Chamber (ft}^3\text{)}}{\text{Rate of Flow to Grit Chamber (ft}^3\text{/sec)}}$$

Clarifiers (pages 138-143)

$$\text{Detention Time (hrs)} = \frac{\text{Volume of Clarifier (gal)}}{\text{Rate of Flow to Clarifier (gal/hr)}}$$

$$\text{Weir Overflow Rate (gal/ft/day)} = \frac{\text{Rate of Flow to Clarifier (gal/day)}}{\text{Length of Weir (ft)}}$$

$$\text{Surface Settling Rate (gal/ft}^2\text{/day)} = \frac{\text{Rate of Flow to Clarifier (gal/day)}}{\text{Surface Area of Clarifier (ft}^2\text{)}}$$

Anaerobic Digesters (pages 144-151)

$$\text{Volatile Acids/Alkalinity Ratio} = \frac{\text{Volatile Acids Analysis (mg/l)}}{\text{Alkalinity Analysis (mg/l)}}$$

$$\text{Digester Loading (lbs/ft}^3\text{/day)} = \frac{\text{Volatile Suspended Solids Fed (lbs/day)}}{\text{Volume digester used (ft}^3\text{)}}$$

PROCESSING FORMULAS

Trickling Filters (pages 152-157)

$$\text{Hydraulic Loading (MGAD)} = \frac{\text{Flow (MGD)} \times 43{,}560\ (ft^2/A)}{\text{Surface Area } (ft^2)}$$

$$\text{Hydraulic Loading } (gal/ft^2/day) = \frac{\text{Flow (gal/day)}}{\text{Surface Area } (ft^2)}$$

$$\text{Organic Loading (B.O.D.) } (lbs/1000ft^3/day) = \frac{\text{B.O.D. to Trickling Filter (lbs/day)}}{\frac{\text{Volume Filter Media } (ft^3)}{1000}}$$

Activated Sludge (pages 158-163)

$$\text{Inf. B.O.D. (lbs)} = \text{B.O.D. Anal. (mg/l)} \times 8.34\ \frac{(lbs/MG)}{mg/l} \times \text{Flow (MG)}$$

$$\text{Inf. S.S. (lbs)} = \text{S.S. Anal. (mg/l)} \times 8.34\ \frac{(lbs/MG)}{mg/l} \times \text{Flow (MG)}$$

$$\text{Wgt MLSS (lbs)} = \text{MLSS Anal. (mg/l)} \times 8.34\ \frac{(lb/MG)}{mg/l} \times \text{Vol. Aer. Tank (MG)}$$

$$\text{Food/Mass Ratio} = \frac{\text{Influent B.O.D. (lbs)}}{\text{MLSS in Aeration Unit (lbs)}}$$

$$\text{Sludge Age (days)} = \frac{\text{Suspended Solids in Aeration Tank (MLSS)(lbs)}}{\text{Suspended Solids in Influent (lbs/day)}}$$

$$\text{Sludge Volume Index (SVI)} = \frac{\text{Settled Sludge Volume (ml/l)} \times 1000}{\text{Suspended Solids (mg/l)}}$$

Sludge Handling (pages 164-167)

$$\text{\% Capture} = \frac{\text{Sludge S.S. (mg/l)} - \text{Return Liquor S.S. (mg/l)}}{\text{Sludge S.S. (mg/l)}} \times 100$$

$$\text{\% Moisture} = \frac{\text{Wgt Wet Sludge (mgs)} - \text{Wgt Dry Solids (gms)}}{\text{Wgt Wet Sludge (gms)}} \times 100$$

PROCESSING FORMULAS

Chlorination (pages 168-172)

$$\text{Dose (mg/l)} = \text{Demand (mg/l)} + \text{Residual (mg/l)}$$

$$\text{Rate of Feed (lbs/day)} = \text{Dose (mg/l)} \times 8.34 \left(\frac{\text{lbs/mg}}{\text{mg/l}}\right) \times \text{Flow (MGD)}$$

$$\text{Detention Time (min)} = \frac{\text{Vol. Tank (gal)} \times 24 \text{ (hrs/day)} \times 60 \text{ (min/hr)}}{\text{Flow (gal/day)}}$$

Percent Efficiency (pages 102-107)

$$\%\ \text{Efficiency (Removal)} = \frac{\text{Input-Output}}{\text{Input}} \times 100$$

$$\%\ \text{Efficiency} = \frac{\text{Influent (mg/l)} - \text{Influent (mg/l)}}{\text{Influent (mg/l)}} \times 100$$

$$\%\ \text{Efficiency} = \frac{\%\ \text{Vol. Solids (Raw)} - \%\ \text{Vol. Solid (Digested)}}{\%\ \text{Vol. Solids (Raw)}} \times 100$$

Laboratory Calculation (pages 108-126)

$$\text{Sewage Solids (mg/l)} = \text{Wgt. of Sample (gms)} \times 1000 \text{ (mg/gm)} \times \frac{1000 \text{ (ml/l)}}{\text{Vol. of sample (ml)}}$$

$$\%\ \text{Sludge Solids} = \frac{\text{Wgt. Dry Sample (gms)}}{\text{Wgt. Wet Sample (gms)}} \times 100$$

$$\%\ \text{Sludge Volatile Solids} = \frac{\text{Wgt. Dry Sample (gms)} - \text{Wgt. Ash (gms)}}{\text{Wgt. Dry Sample (gms)}} \times 100$$

$$\text{B.O.D. (mg/l)} = \text{Initial D.O. (mg/l)} - \text{Final D.O. (mg/l)} \times \frac{1000 \text{ (ml/l)}}{\text{Vol. of sample (ml)}}$$

$$\text{Init. Corr. D.O.} = \text{Initial D.O. (mg/l) dilution water} - \frac{1}{10}\left(\text{Initial D.O. (mg/l) dilution water} - \text{Final D.O. (mg/l) Seed Sample}\right)$$

ANSWER SHEETS

ADDITION OF WHOLE NUMBERS

1) 10
2) 15
3) 9
4) 13
5) 8
6) 8
7) 16
8) 20
9) 18
10) 23
11) 23
12) 24
13) 29
14) 1,632
15) 991
16) 572
17) 1256

SUBTRACTION OF WHOLE NUMBERS

1) 7
2) 3
3) 8
4) 5
5) 7
6) 6

SUBTRACTION OF WHOLE NUMBERS CONTINUED

7) 6
8) 5
9) 2
10) 275
11) 1,179
12) 2,102

MULTIPLICATION OF WHOLE NUMBERS

1) 12
2) 15
3) 24
4) 270
5) 90
6) 1,400
7) 400
8) 900
9) 40
10) 1,122
11) 6,768
12) 30,702
13) 15,708
14) 6,688
15) 56, 088
16) 177,984
17) 629,107

DIVISION OF WHOLE NUMBERS

1) 15 R5
2) 1,218 R4
3) 1,001 R2
4) 855
5) 24
6) 248 R20
7) 24 R25
8) 256 R8

WHOLE NUMBER REVIEW QUESTIONS

1) 1462
2) 230
3) 791
4) 42,294
5) 428

FRACTIONS AND FRACTIONAL NUMBERS

1) $\frac{1}{3}$

2) $\frac{1}{4}$

3) $\frac{3}{16}$

4) 1

5) 2

6) $5\ \frac{1}{5}$

7) $\frac{1}{2}$

8) $\frac{1}{6}$

9) $2\ \frac{7}{10}$

10) $3\ \frac{1}{2}$

11) 3

12) 25

ADDITION OF FRACTIONS

1) $\frac{11}{8} = 8/\overline{11}$ $\quad \frac{1 \text{ R3}}{8} = 1\ \frac{3}{8}$

2) $\frac{12 \div 4}{16 \div 4} = \frac{3}{4}$

3) $\frac{11}{12}$ (already simplest form)

4) 12

5) 12

6) $\frac{7}{6}$ or $1\ \frac{1}{6}$

7) $\frac{17}{24}$

ADDITION OF FRACTIONS CONTINUED

8) $\frac{7}{36}$

9) $\frac{22}{15}$ or $1\ \frac{7}{15}$

10) $\frac{20}{24}$ or $\frac{5}{6}$

11) $\frac{26}{30}$ or $\frac{13}{15}$

12) $93\ \frac{13}{36}$

13) $63\ \frac{5}{6}$

SUBTRACTION OF FRACTIONS

1) $\frac{10}{16}$ or $\frac{5}{8}$

2) $\frac{3}{13}$

3) $\frac{38}{50}$ or $\frac{19}{25}$

4) $\frac{11}{16}$

5) $\frac{17}{24}$

6) $3\ \frac{7}{20}$

7) $41\ \frac{5}{24}$

8) $8\ \frac{6}{7}$

9) $4\ \frac{7}{16}$

10) $10\ \frac{14}{15}$

MULTIPLICATION OF FRACTIONS

1) $\frac{9}{11}$

2) $2\frac{1}{7}$

3) $\frac{1}{2}$

4) $\frac{18}{25}$

5) $\frac{2}{9}$

6) $\frac{40}{63}$

7) 6

8) $2\frac{2}{15}$

9) $\frac{1}{6}$

DIVISION OF FRACTIONS

1) $\frac{9}{22}$

2) $\frac{3}{5}$

3) $\frac{8}{35}$

FRACTION REVIEW QUESTIONS

1) $1\frac{5}{8}$

2) $3\frac{4}{7}$

3) $3\frac{1}{2}$

4) $2\frac{1}{3}$

FRACTION REVIEW QUESTIONS CONTINUED

5) $\frac{1}{3}$

6) $\frac{1}{3}$

7) $\frac{2}{3}$

8) $\frac{88}{21} = 4\frac{4}{21}$

9) $1\frac{3}{10}$

10) $2\frac{3}{4}$

11) $\frac{67}{96}$

12) $\frac{2}{9}$

13) $\frac{1}{60}$

14) $\frac{13}{300}$

15) $\frac{3}{35}$

16) $2\frac{1}{10}$

17) $\frac{15}{28}$

18) $\frac{5}{21}$

19) $\frac{5}{21}$

20) $2\frac{1}{7}$

ADDITION OF DECIMALS

1. 37.2
2. 48.8472
3. 18.6

SUBTRACTION OF DECIMALS

4. 1.9
5. 33.669
6. 13.079

MULTIPLICATION OF DECIMALS

7. 430
8. 3.5
9. 35,610.
10. 7.40235
11. 6.57
12. 30.069

DIVISION WITH DECIMAL NUMERALS

13. .643
14. .0146
15. .004
16. 4.651
17. 4.98
18. 30.5
19. 4.18
20. 397.
21. 3.17
22. 404
23. .875
24. .28

DIVISION WITH NUMERALS CONTD.

25. .0064
26. $\frac{1}{5}$
27. $\frac{3}{4}$
28. $\frac{4}{10}$ or $\frac{2}{5}$

PERCENTAGE

1. $\frac{35}{100}$
2. $\frac{15}{100}$
3. $\frac{71}{100}$
4. $\frac{50}{100} = \frac{1}{2}$
5. $\frac{10}{100} = \frac{1}{10}$
6. $\frac{80}{100} = \frac{4}{5}$
7. $\frac{131}{100} = 1\ \frac{31}{100} = 1.31$
8. $\frac{245}{100} = 2\ \frac{45}{100} = 2\ \frac{9}{20} = 2.45$
9. $\frac{161}{100} = 1\ \frac{61}{100} = 1.61$
10. .21
11. 790%
12. 35%
13. .0023
14. $\frac{9}{20}$
15. $1\ \frac{1}{10}$

PERCENTAGE CONTINUED

16. $\frac{3}{20}$

17. $\frac{7}{100}$

18. .43

19. .035

20. .2

21. .36

22. 15.7%

23. 380%

24. 12%

25. 9%

26. $3.00

27. 64 days

RATIO AND PROPORTION

1. 18

2. 49

3. 250 lbs.

LINES AND DISTANCES

1. 2 in.
2. $2\frac{3}{4}$ in.
3. $4\frac{1}{8}$ in.
4. 9 cm or 90 mm
5. 7.9 cm or 79 mm
6. 11.5 cm or 115 mm

PERIMETERS AND AREAS

1. 30 yds
2. 17 ft.
3. 11 mi
4. 15 ft^2
5. 6 yds^2
6. 7 in^2
7. 24 ft
8. 12 yds
9. 10 in
10. 37 ft
11. 49 in^2
12. 441 yd^2
13. 10 in^2

PERIMETER & AREAS (cont'd)

14. 90 ft^2
15. 7 in^2
16. 240 ft^2
17. 44 ft
18. 37.68 ft
19. 907.46 in^2
20. 113.04 ft^2

VOLUMES

1. 343 ft^3
2. 8 yd^3
3. 30 yds^3
4. 72 ft^3
5. 441 ft^3
6. 81 ft^3
7. 863.5 ft^3
8. 50868 ft^3

BASIC MATHEMATICS REVIEW PROBLEMS

1. 1462
2. 791
3. 136746
4. 2
5. 144 ft^2
6. 343 in^3
7. $1\frac{5}{8}$
8. $3\frac{1}{2}$
9. $2\frac{1}{3}$
10. $\frac{1}{3}$
11. $\frac{2}{3}$
12. $1\frac{13}{30}$
13. $\frac{2}{3}$
14. $\frac{1}{60}$
15. $\frac{13}{300}$
16. $\frac{3}{35}$
17. $\frac{5}{21}$
18. .0833
19. 0.875
20. $\frac{3}{4}$
21. $\frac{19}{20}$
22. 17.836
23. 7.31
24. 5.5883
25. 0.73
26. 33.22111
27. 11.714859
28. 20.842066
29. 0.084
30. $\frac{17}{100}$
31. $\frac{1}{30}$
32. .0225
33. .16
34. 80%
35. 62.5%
36. 100
37. 30
38. (a) 56 ft
    (b) 160 ft^2
39. (a) 125.6 ft
    (b) 1256
40. 110 ft^2
41. 216 ft^3
42. 1500 ft^3
43. 2200 ft^3
44. (a) 1526 ft^3
    (b) 169.6 ft^3

PERCENT EFFICIENCY

1) (a) 50 %
   (b) 50 %
   (c) 66.7 %
   (d) 33.3 %
   (e) 94.4 %

LABORATORY CALCULATIONS

Sewage Solids

1) (a) 0.0062 gms
   (b) 6.2 mgs
   (c) 155 mg/l

2) 74.2 %

Sludge Solids

1) (a) % Solids = 7.3 %
   (b) % Volatile Solid = 74.0 %

2) (a) % Solids = 2.2 %
   (b) % Volatile Solids = 36.4 %

B. O. D.

1) (a) 15 ml factor = 20
   10 ml factor = 30
   5 ml factor = 60

   (b) 15 ml Depletion = 6.2 mg/l
   10 ml Depletion = 4.4 mg/l
   5 ml Depletion = 2.0 mg/l

   (c) 15 ml B.O.D. = 124 mg/l
   10 ml B.O.D. = 132 mg/l
   5 ml B.O.D. = 120 mg/l

   (d) B.O.D. = 125 mg/l

2. Corrected Initial D.O. = 7.6

GRIT CHAMBER

1) 1.25 ft/sec

2) (a) 3.596 or 3.6 ft^3/sec
   (b) 4.5 ft^2
   (c) 0.8 ft/sec

3) (a) 560 ft^3
   (b) 420 ft^3
   (c) 980 ft^3
   (d) 9.765 or 9.8 ft^3/sec
   (e) 100 sec or 1 $\frac{2}{3}$ min

CLARIFIERS

1) (a) 4906 ft^3
   (b) 36,697 gal.
   (c) 20,833 gal/hr
   (d) 1.76 hrs.
   (e) 75.4 ft.
   (f) 6631 gal/ft/day
   (g) 1962.5 ft^2
   (h) 255 gal/ft^2/day

ANAEROBIC DIGESTION

1) (a) 4,170 lbs.
   (b) .087 or .09 lbs/ft^3/day

2) 0.15

3. (a) 44.7 %
   (b) 16.7 %
   (c) 53.9 %

TRICKLING FILTERS

1) (a) 7085 ft^2
   (b) 706 gal/ft^2/day

2) (a) 54,047 ft^3
   (b) 54.047 1000 ft^3
   (c) 125 lbs/1000 ft^3/day

ACTIVATED SLUDGE

1) (a) 100,000 ft^3
   (b) 748,000 gal.
   (c) 4,378.5 lbs.
   (d) 6,255 lbs.
   (e) 18,715 lbs.
   (f) 0.234
   (g) 3 days

SLUDGE HANDLING

1) (a) 105,000 mg/l
   (b) 55 %

2) 77.8 %

PROCESSING REVIEW PROBLEMS

1) 0.75 ft/sec

2) (a) 240 ft^3
(b) 480 ft^3
(c) 720 ft^3
(d) 5,385.6 gal.

3) (a) 109.9 ft
(b) 113.0 ft
(c) 223 ft
(d) 7,623 gal/ft/day

4. 0.08

5) (a) 458.7 lbs
(b) 11.9 lbs/1000 ft^3/day

6) (a) 75,000 mg/l
(b) 80 %

7) (a) 4.0 mg/l
(b) 200.16 lb/day

8) 155 mg/l

9) (a) 14.4 %
(b) 69.4 %

BASIC MATHEMATICS & WASTEWATER PROCESSING CALCULATIONS
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CORRECTION SHEET

Page 72 — Example of Volume of Trapezoidal Solid

Answer given is incorrect: $V = 840\ ft^3$

Correct answer is: $V = 630\ ft^3$

Page 122 — Laboratory Analysis Sheet for determining the BOD of a sample. Under the column titled "Seed Sample".

Initial Burette Reading valve is incorrect: 28.2

Correct Initial Burette Reading is: 28.0

Page 135 — Example of determination of Grit Chamber Detention Time (Step 3)

Answer given is incorrect: D.T. = 53.5 sec

Correct answer is: D.T. = 52.9 sec

Page 149 — Example of determination of Digester Loading (Step 3)

Answer given is incorrect: Digester Loading = 0.78 $lbs/ft^3/day$

Correct answer is: Digester Loading = 0.078 $lbs/ft^3/day$

Page 159 — Example of determination of weight of BOD (lbs)

Answer given is incorrect: weight of BOD (lbs) = 13,280 lbs.

Correct answer is: weight of BOD (lbs) = 13,344 lbs.

Page 185 — Processing Formulas, under percent efficiency

Add the following formula for Anaerobic Digesters:

$$\text{Digester Efficiency} = \frac{\%\ \text{V.S. (raw)} - \%\ \text{V.S. (Digester Supernatant)}}{\%\ \text{V.S. (raw)} - (\%\ \text{V.S. (raw)} \times \%\ \text{V.S. (supernatant)})} \times 100$$

(% V.S. Reduction)

Page 193 — Answers to Problems in "Percent Efficiency" Section

These appear as: 1) (a) 50%
(b) 50%
(c) 66.7%
(d) 33.3%
(e) 94.4%

They should appear as: 1) 59.3%

2) (a) 50%
(b) 50%
(c) 66.7%
(d) 33.3%
(e) 94.4%

Page 194

Answers to Problems in Anaerobic Digestion Section
(Problem # 3 on page 151)

Answers for Problem #3 are incorrect:

3) (a) 44.7%
(b) 16.7%
(c) 53.9%

Correct answers are:

3) (a) 77.1%
(b) 25.6%
(c) 83%

Delete Example #2 on pages 103 and 104. (This example is incorrect.)

Consider Example #3 on page 104 as Example #2

Add pages 106A and 106B (containing new Example #3)

Also add on page 183 the Digester Efficiency formula:

$$\text{Digester Efficiency (\% V.S. Reduction)} = \frac{\text{\% V.S. (raw)} - \text{\% V.S. (digester supernatent)}}{\text{\% V.S. (raw} - (\text{\$ V.S. (raw)} \times \text{\% V.S. (supernatent)})} \times 100$$

NOTE: V.S. Means Volatile Solids

Example # 3

In the previous examples we were concerned with efficiencies within the water flow through a plant. In calculating the efficiency of removals in an anaerobic digester, we are concerned with the solids stream. Because of the significant changes in total volume and weight of the solids stream adjustments must be made in the percent of solids remaining after digestion. For example, let's assume a raw primary sludge containing 70% volatile matter was fed to a digester. The digested sludge contained 50% volatile matter.

The basic efficiency equation would result in the following:

$$\frac{\text{Input-Output}}{\text{Input}} \times 100 = \frac{.70 - .50}{.70} \times 100 = 28.6\%$$

However, a mass balance will show that this is incorrect.

Before Digestion	After Digestion
Assume 100 lbs. sludge	Total = 60 lbs. sludge
70% = 70 lbs. volatile matter	50% volatile = 30 lbs. volatile
30% = 30 lbs. inert matter	50% inert = 30 lbs. inert

This is true because the same amount of inert material will be in the digested sludge as was in the raw sludge. Therefore, 70 lbs. - 30 lbs. = 40 lbs. volatile matter destroyed.

Or: $\frac{\text{Input-Output}}{\text{Input}} \times 100 = \frac{70 \text{ lbs.} - 30 \text{ lbs.}}{70 \text{ lbs.}} \times 100 = 57.1\%$

An equation has been developed which precludes the necessity of doing a mass balance.

$$\frac{\text{Input-Output}}{\text{Input-(Input x Output)}} \times 100 = \% \text{ Volatile matter destroyed in an anaerobic digester}$$

$$\frac{.70 - .50}{.70 - (.7 \times .5)} \times 100 = 57.1\% \text{ Volatile matter destroyed}$$

106B

Problem # 1

A total of 1000 lbs. of raw primary sludge containing 75% volatile matter is fed to an anaerobic digester each day. The digested sludge contains 55% volatile matter.

a) What is the percentage of volatile matter destroyed?

Sewage

Treatment

Plant

Operators

T E N C O M M A N D M E N T S

1. *Thou shalt be pipe-fitter, electrician, office boy, carpenter, or what-have-you, for the sake of the sewage pumping plant.*

2. *Thou shalt not take in vain the name of thy superintendent, nor of the commissioners.*

3. *Five days shalt thou labor with thy plant and on the sixth day and seventh days, thou shalt labor also, for Saturday and Sunday were made for poor fools like thee.*

4. *Thou shalt keep the plant running on Election Day, Fourth of July, Labor Day, Thanksgiving, Christmas and New Year's Day. For the welfare of the fair county, thou shalt have no holidays whatsoever.*

5. *Thou shalt operate the plant without residues, either those that pollute with stenches the air above, or those that may be poured into river. Thou shalt do this without asking others how it is done.*

6. *Thou shalt endeavor by hydraulics, politics, electronics or softsoap, bull-dozing profanity and chemistry to keep their efficiencies high - yes, 100% is acceptable.*

7. *Thou shalt not covet new equipment, or helpers, or assistants, for all these things are an abomination in the sight of the tax-payers.*

8. *Thou shalt never, no never, assign any one of the laborers, special laborers or plant operators to any menial task which he might consider beneath his dignity, or in any manner discriminatory.*

9. *Thou shalt maintain continuously, and at all times, full operation with no sewage entering water-ways.*

10. *Thou shalt accept gratefully and without question the pittance meted out to thee. May thy troubles be many, thy financial and spiritual reward small.*

"Anonymous"

...revised and adapted by Domenic Rossi, Supervising Operator, North Yonkers Pumping Station, County of Westchester, New York.